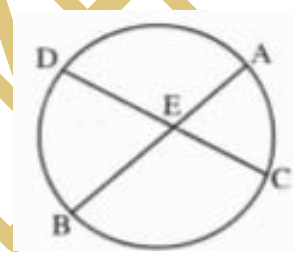
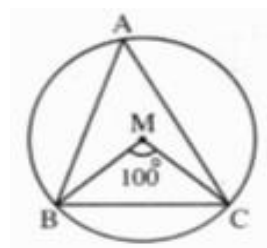
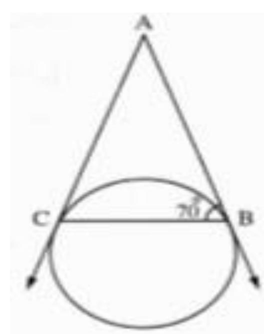
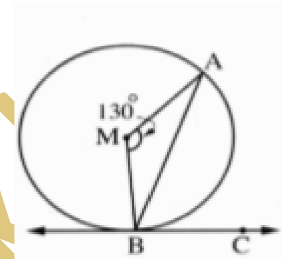
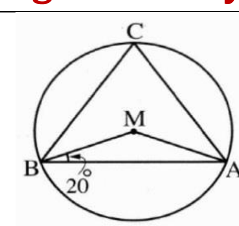
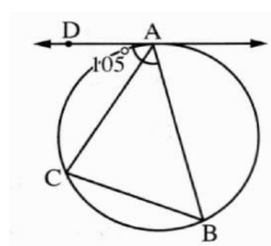
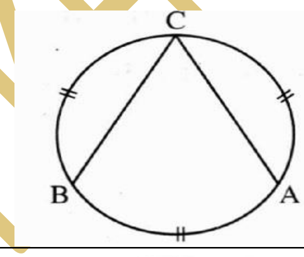
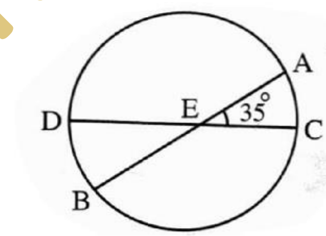
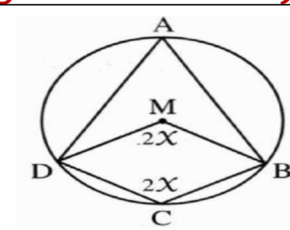
PREP 3 **FINAL REVISION**

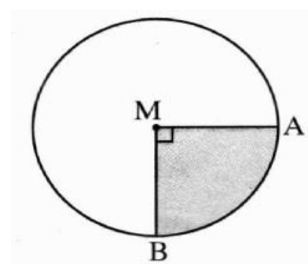
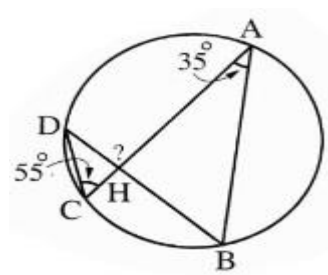
SECOND GEOMETRY

Choose the correct answer:

1)	In the opposite figure: m $\overset{\frown}{(DB)} = 80^{o}$, m $\overset{\frown}{(AC)} = 60^{o}$, then m $(\angle AEC) =$ ……….  (20° or 30° or 70° or 140°)
2)	The two tangents which are drawn from the two endpoints of a diameter of a circle are ……….. (parallel. or intersecting. or perpendicular. or coincide.)
3)	In the opposite figure: M is a circle , m $(\angle BMC) = 100°$, then m $(\angle BAC) =$ ……….  (150° or 100° or 50° or 25°)
4)	In the opposite figure: $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C , m $(\angle ABC) = 70°$, then m $(\angle A) =$ ……….  (140° or 70° or 40° or 35°)
5)	Sum of the measures of any two opposite angles in the cyclic quadrilateral equals …………. (90° or 180° or 270° or 360°)

6)	Measure of an are which represents $\frac{1}{3}$ of the measure of the circle equals = ………… (60° or 90° or 120° or 180°)
7)	In the opposite figure: $\overrightarrow{BC}$ is a tangent to the circle M at B if m $(\angle AMB) = 130°$, than m $(\angle ABC) =$ ………… (280° or 140° or 70° or 65°) 
8)	The length of the arc which represents $\frac{1}{4}$ of circumference of a circle = ………… ($2\pi r$ or πr or $\frac{1}{2}\pi r$ or $\frac{1}{4}\pi r$)
9)	In a cyclic quadrilateral, each two opposite angles are ………. equal or supplementary intersecting or corresponding
10)	If surface of circle M ∩ surface of circle N = ∅ , than the two circles are …………. intersecting or distant touching internally or touching externally
11)	In the opposite figure: Circle M , if m $(\angle MBA) = 20°$, then m $(\angle C) =$ ………… (120° or 70° or 40° or 30°) 
12)	In the opposite figure: If $\overleftrightarrow{AD}$ is a tangent to the circle at A , m $(\angle DAB) = 105°$, then m $(\angle ACB) =$ ………… (75° or 60° or 50° or 35°) 

13)	The number of common tangents for the two tangent circles externally is .... (4 or 3 or 2 or infinite number)
14)	The figure which the circle doesn't passing through its vertices is .... (square or rectangle or rhombus or triangle)
15)	In the opposite figure: $m(\angle C) =$ .... (45° or 50° or 30° or 60°) 
16)	In the opposite figure: $(\angle AEC) = 35°$, then $m(\overset{\frown}{AC}) + m(\overset{\frown}{DB}) =$ .... (17.5° or 35° or 70° or 140°) 
17)	The inscribed angle opposite to an arc greater than the semicircle is .... (straight or acute or right or obtuse)
18)	In the opposite figure: If $m(\angle DMB) = m(\angle DCB) = 2x$, than $m(\angle A) =$ .... (60° or 70° or 40° or 30°) 
19)	The diameter length of a circle is 8 cm. if the straight line L is at a distance 4 cm. form the Centre , then the straight line L is .... a secant to the circle. or outside the circle. a tangent to the circle. or an axis of symmetry to the circle.

20)	The measure of the exterior angle at any vertex of a cyclic quadrilateral vertices........... the measure of the opposite interior of the adjacent angle. (> or < or = or ≥)
21)	The number of common tangents of two distant circles is .... (4 or 3 or 2 or infinite)
22)	The length of the arc opposite to the inscribed angle of measure .60° = ... Circumference of the circle. ($\frac{1}{6}$ or $\frac{1}{3}$ or $\frac{1}{2}$ or otherwise)
23)	The inscribed angle drawn in a semicircle .... (acute or obtuse or reflex or right)
24)	In the opposite figure: $\overline{MA}$ and $\overline{MB}$ two radii in a circle M , $\overline{MA} \perp \overline{MB}$ and the radius length is 7 cm. then the perimeter of the shaded part =...........cm. (14 or 21 or 38.5 or 25) 
25)	The measure of the circle with radius r is .... ($2\pi r$ or 180^o or πr or 360^o)
26)	In the opposite figure: m (∠ C) = 55° , m (∠ A) = 35° , then m (∠ AHD) = ... (20^o or 90^o or 70^o or 110^o) 
27)	The Centre of inscribed circle of a triangle is the intersection point of is .... altitudes. or axes of symmetry of its sides. medians. or bisectors of its interior angles.

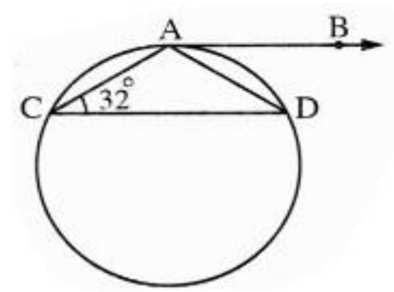
28) In the opposite figure:

$\overrightarrow{AB}$ is tangent to the circle,

m (∠ C) = 32°

, then m (∠ BAD) = ...

(64° or 32° or 148° or 58°)



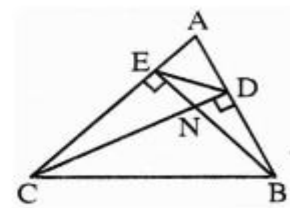
29) If M and N are two touching externally circles with radii lengths 9 cm. and r cm. respectively , if MN = 14 cm. , then r = ... cm.

(10 or 23 or 5 or 7)

30) In the opposite figure:

How many cyclic quadrilaterals?

(1 or 2 or 3 or 4)



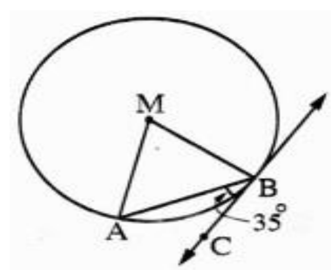
31) In the opposite figure:

$\overleftrightarrow{BC}$ is a tangent to the circle M

, m (∠ ABC) = 35°

, then m (∠AMB) = ...

(105° or 120° or 70° or 60°)

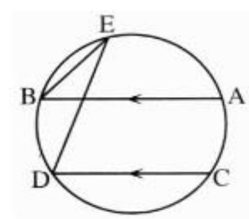


32) In the opposite figure:

$\overline{AB}$ and $\overline{CD}$ are two parallel chords of a circle ,

m(∠DEB) = 25° , then m $(\overset{\frown}{AC})$ = ...

(100° or 75° or 50° or 25°)



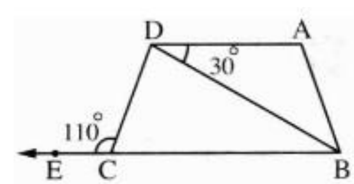
33) In the opposite figure:

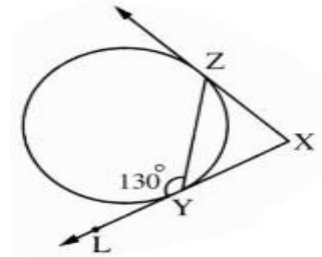
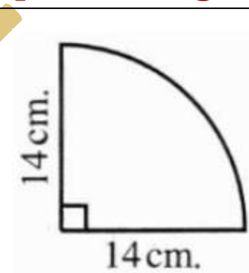
ABCD is a cyclic quadrilateral ,

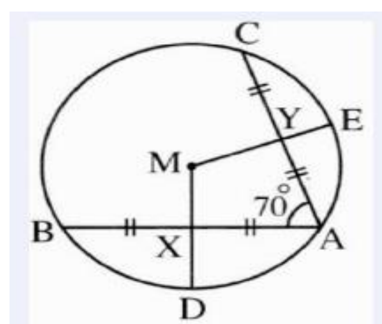
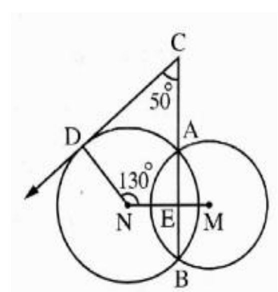
m (∠ ADB) = 30°

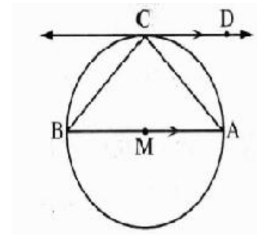
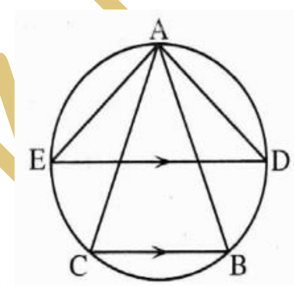
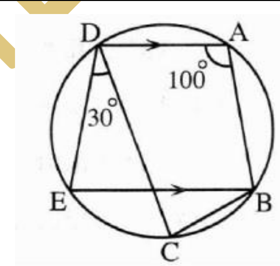
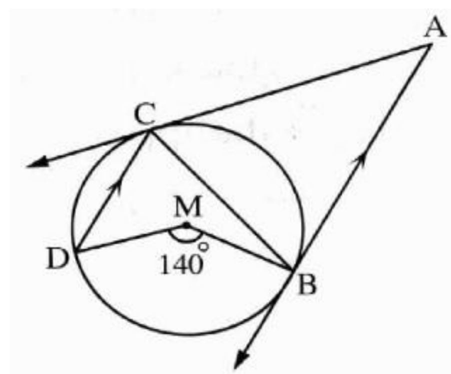
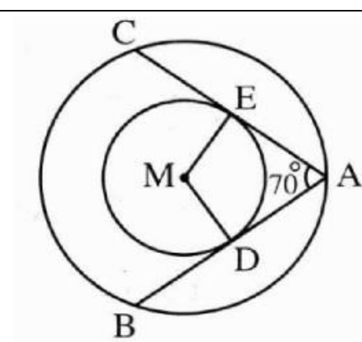
and m (∠DCE) = 110° , then m (∠ABD) = ...

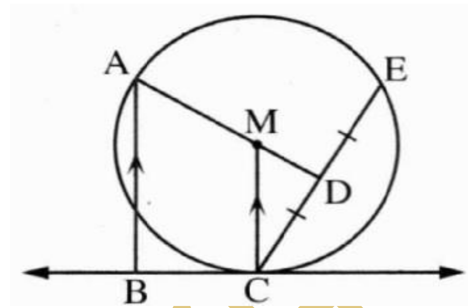
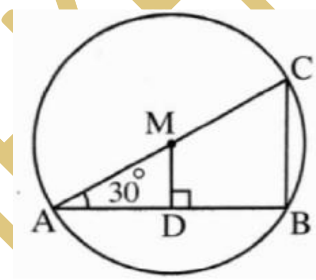
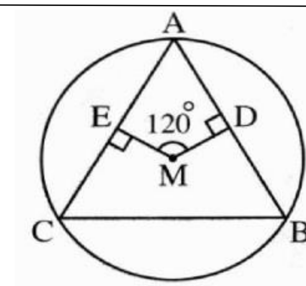
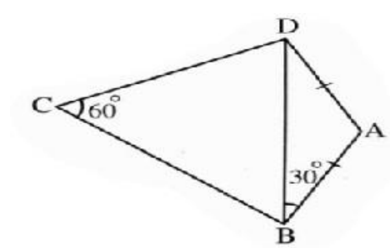
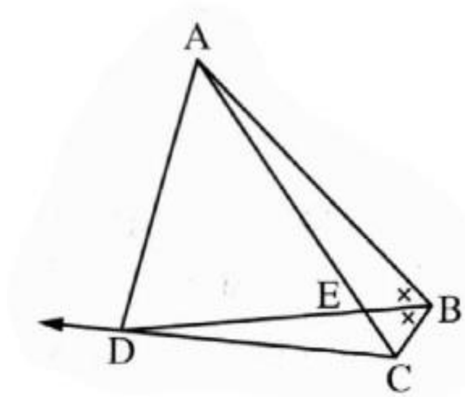
(30° or 40° or 60° or 70°)



34)	In the opposite figure: $\overrightarrow{XZ}$, $\overrightarrow{XL}$ are two tangents to the circle at Y and Z , m (∠ LYZ) = 130° , then m (∠ X) = ... (50° or 65° or 80° or 100°) 
35)	If the measures of the two arcs are equal in the same circle then their chords are .... intersecting. or parallel perpendicular. or equal in length.
36)	In the opposite figure: A metallic wire is formed in the form of a quarter of a circle of radius length 14 cm. as shown , then the length of the wire = ... where $\pi = \frac{22}{7}$ (154 cm. or 50 cm. or 26 cm. or 22cm.) 

1)	a) In the opposite figure: $\overline{AB}$ and $\overline{AC}$ are two equal chords in length In the circle M , X is the midpoint of $\overline{AB}$ and Y Is the midpoint of $\overline{AC}$, m (∠ CAB) = 70° (1) Calculate : m (∠ DME) (2) Prove that: XD = YE  b) In the opposite figure: M and N are two circles intersecting at A and B. and C ∈ $\overrightarrow{BA}$, D ∈ the circle N, m (∠ MND) = 130° , m (∠ BCD) = 50° , Prove that : $\overrightarrow{CD}$ is a tangent to the circle at D 

2)	a) In the opposite figure: $\overline{CD}$ is a tangent to the circle M at C, $\overrightarrow{CD} // \overline{BA}$ Prove that : m (∠ DCA) = 45° 
	b) In the opposite figure: $\overline{DE} // \overline{BC}$ Prove that M (∠ DAC) = m (∠ BAE) 
3)	a) In the opposite figure: $\overline{AD} // \overline{BE}$,m (∠ BAD) = 100° And m (∠ CDE) = 30° Find: m (∠ ADC) 
	b) In the opposite figure $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M at B and C $\overline{AB} // \overline{CD}$, m (∠ BMD) = 140° Find: m (∠ A) 
4)	a) In the opposite figure: Two concentric circles at M , $\overline{AB}$ and $\overline{AC}$ are two tangent segments to the smaller circles, m (∠ A) = 70° (1) Find: m (∠ DME) (2) Prove that : AB = AC 

<table>
<tr><td></td><td>b) In the opposite figure:
$\overleftrightarrow{BC}$ is a tangent to the circle M at C
D is the midpoint of $\overline{EC}$, $\overline{MC} // \overline{AB}$
Prove that : ABCD is a cyclic quadrilateral.

</td></tr>
<tr><td>5)</td><td>a) In the opposite figure:
A circle of Centre M , $\overline{MD} \perp \overline{AB}$,
If m (∠ A) = 30°
(1) Prove that : $\overline{MD} // \overline{CB}$
(2) Find : m (∠ C)

</td></tr>
<tr><td></td><td>b) In the opposite figure:
A circle M , $\overline{MD} \perp \overline{AB}$,
$\overline{ME} \perp \overline{AC}$ where MD = ME ,
m (∠ DME) = 120°
Prove that : the triangle ABC is equilateral.

</td></tr>
<tr><td>6)</td><td>a) In the opposite figure:
If: AB = AD , m (∠ ABD) = 30° ,
m (∠ C) = 60°
Prove that : ABCD is a cyclic quadrilateral

</td></tr>
<tr><td></td><td>b) In the opposite figure:
ABCD is a cyclic quadrilateral, $\overrightarrow{BD}$ bisects ∠ ABC,
If $\overline{BD} \cap \overline{AC}$ = {E}
Prove that : $\overrightarrow{CD}$ is a tangent to the circle Passing through the vertices of Δ BEC

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</table>

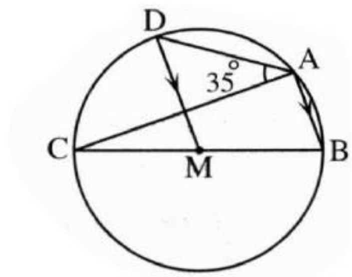
7)

a) In the opposite figure:

$\overline{BC}$ is a diameter in the circle M ,

m (∠ CAD) = 35°, $\overline{AB} // \overline{DM}$,

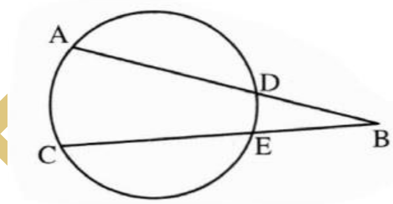
Find: m (∠ ABC)



b) In the opposite figure:

If m $(\overset{\frown}{AC})$ = 120° , m $(\overset{\frown}{DE})$ = 50°

Find: m (∠ ABC)



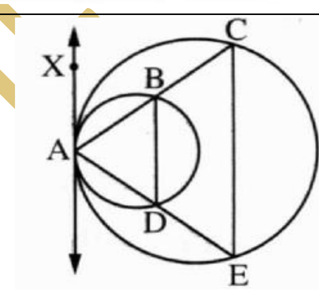
8)

a) In the opposite figure:

If $\overleftrightarrow{AX}$ is a common tangent

to the two circles at A.

Prove that: $\overline{BD} // \overline{CE}$



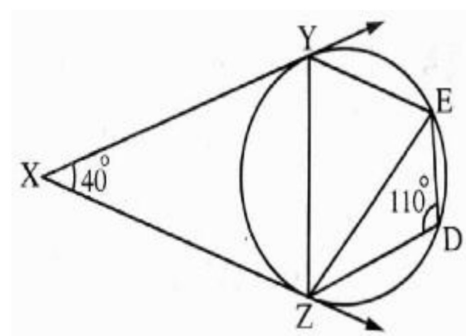
b) In the opposite figure:

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents

to the circle from the point X at Y , Z

, if m (∠ EDZ) = 110°, m (∠ YXZ) = 40°

Prove that : m $(\overset{\frown}{ZDE})$ = m $(\overset{\frown}{ZY})$



9)

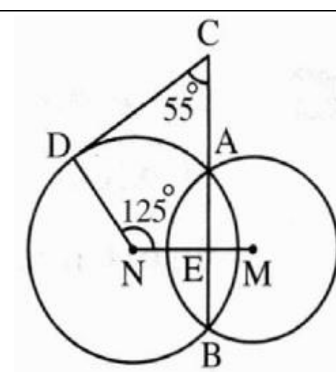
a) In the opposite figure:

M and N are two intersecting circles at A and B

, C ∈ $\overrightarrow{BA}$, D ∈ the circle N

, m (∠ MND) = 125° and m (∠ BCD) = 55°

Prove that: $\overleftrightarrow{CD}$ is a tangent to circle N at D



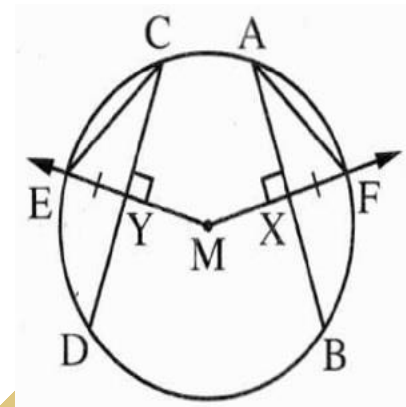
b) In the opposite figure:

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle in F

, $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E

where FX = EY



Prove that: (1) AB = CD

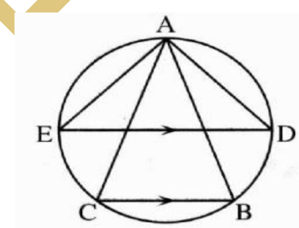
(2) AF = CE

10) a) In the opposite figure:

ABC is an inscribed triangle inside a circle

, $\overline{DE} // \overline{BC}$

Prove that: m (∠ DAC) = m (∠ BAE)



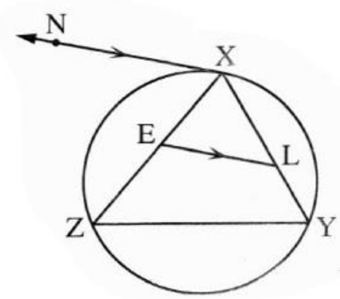
b) In the opposite figure:

XYZ is an inscribed triangle in a circle

, $\overline{LE}$ paralled tangent $\overrightarrow{XN}$

Prove that :

LYZE is cyclic quadrilateral.



11) a) In the opposite figure:

$\overline{AB}$, $\overline{AC}$ are two tangents

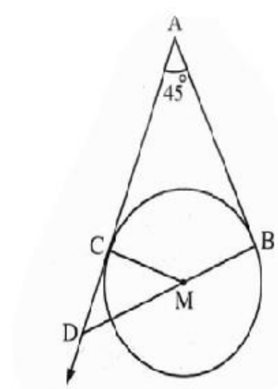
To circle M at B , C ,

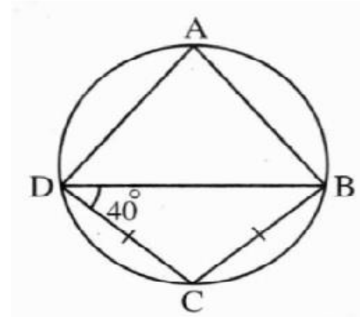
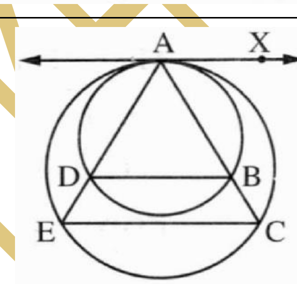
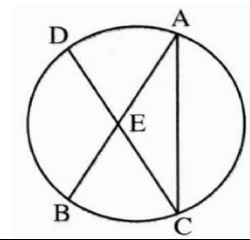
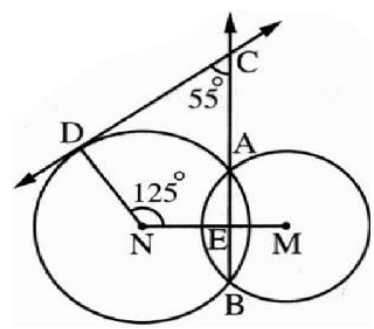
m (∠ A) = 45°

Prove that:

(1) ABMC is cyclic quadrilateral.

(2) AD = AB + MB



	b) In the opposite figure: ABCD is a quadrilateral inscribed in circle , BC = CD , m (∠ BDC) = 40° Find: m (∠ A) 
12)	a) In the opposite figure: Prove that: $\overline{BD} // \overline{CE}$  b) In the opposite figure: $\overline{AB}$, $\overline{CD}$ are two equal chords in length Prove that : the triangle ACE is an isosceles triangle. 
13)	a) In the opposite figure: M and N are two intersecting circles at A and B , C ∈ $\overrightarrow{BA}$ and D ∈ the circle N , m (∠ MND) = 125°, m(∠ BCD) = 55° Prove that: $\overleftrightarrow{CD}$ is a tangent to the circle N at D 
	b) $\overline{AB}$ and $\overline{CD}$ are two chords in the circle M , $\overrightarrow{MX}$ is drawn perpendicular to $\overline{AB}$ to intersect the circle in F and $\overrightarrow{MY}$ is drawn perpendicular to $\overline{CD}$ to intersect the circle at E , if FX = EY Prove that : (1) AB = CD (2) AF = CE

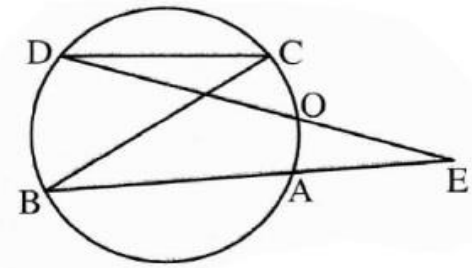
14)

a) In the opposite figure:

E is a point outside the circle

Prove that :

$m(\angle DCB) > m(\angle E)$

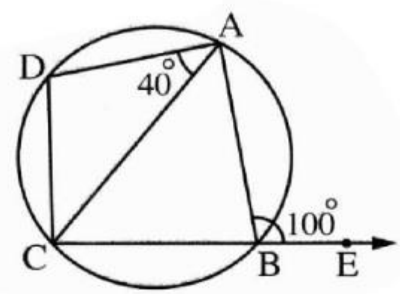


b) In the opposite figure:

$m(\angle ABE) = 100°$

, $m(\angle CAD) = 40°$

Prove that:
$m(\overset{\frown}{CD}) = m(\overset{\frown}{AD})$



15)

Complete:

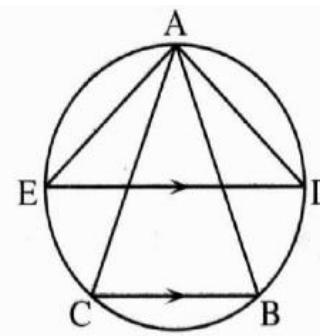
a) The straight line passing through the center of the circle and the intersection point of the two tangents are ………… to the chord of tangency of those two tangents.

b) In the opposite figure:

ABC is an inscribed triangle inside the circle

, $\overline{DE} // \overline{BC}$

Prove that : $m(\angle DAC) = m(\angle BAE)$

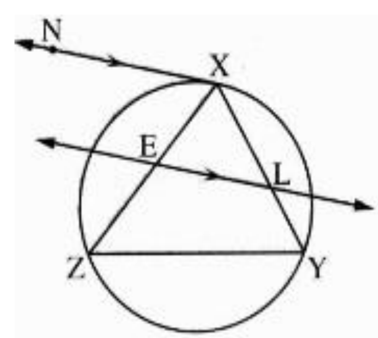


16)

In the opposite figure:

XYZ is an inscribed triangle in a circle , if $L \in \overline{XY}$ and $\overleftrightarrow{LE}$ is drawn parallel to the tangent $\overrightarrow{XN}$ which touches the circle at X and intersects $\overline{XZ}$ at E

Prove that : LYZE is a cyclic quadrilateral.

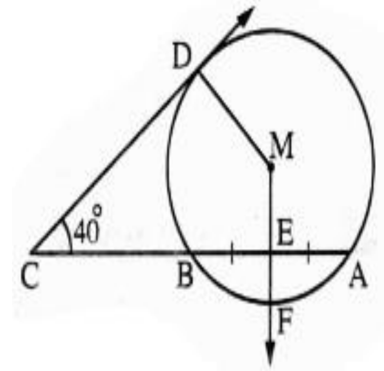


17) a) In the opposite figure:

A circle M whose radius length is 10 cm., m (∠ DCA) = 40° , AB = 16 cm.

, E is the midpoint of $\overline{AB}$, $\overrightarrow{CD}$ is a tangent to the circle

Find by proof : m (∠ DMF) , the length of $\overline{FE}$

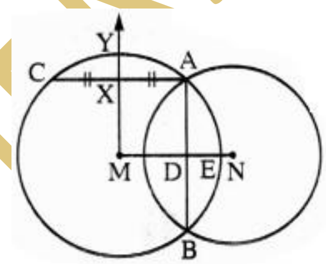


b) In the opposite figure:

If M and N are two intersecting circles at A and B , AB = AC ,

X is the midpoint of $\overline{AC}$

Prove that : XY = DE



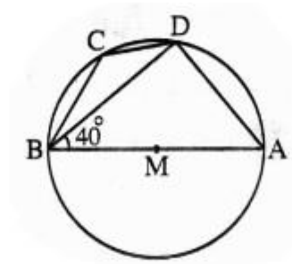
18) a) In the opposite figure:

$\overline{AB}$ is a diameter of circle M ,

m (∠ ABD) = 40°

Find

m (∠ A) , m (∠ C)

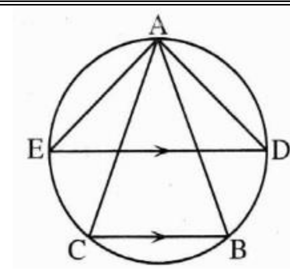


19) b) In the opposite figure:

ABC is an inscribed triangle in the circle ,

$\overline{ED}$ // $\overline{BC}$

Prove that: m (∠ DAC) = m (∠ BAE)

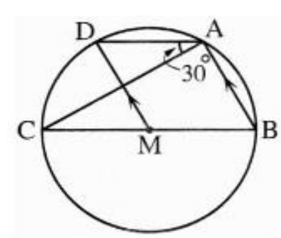


20) a) In the opposite figure:

$\overline{CB}$ is a diameter of circle M ,

$\overline{AB}$ // $\overline{DM}$, m (∠ DAC) = 30°

Find: m (∠ ACB)

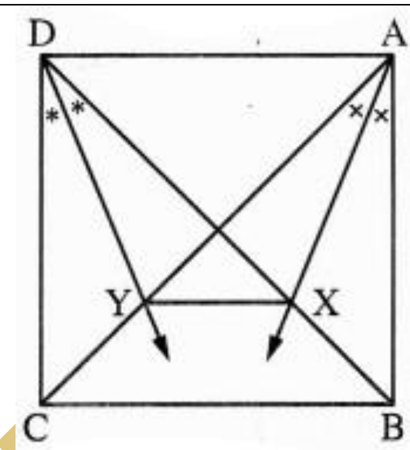


b) In the opposite figure:

ABCD is a square , $\overrightarrow{AX}$ bisects ∠ BAC and $\overrightarrow{DY}$ bisects ∠ CDB

(1) Prove that the figure AXYD is cyclic quadrilateral

(2) Find with proof m (∠ DXY)



21) In the opposite figure:

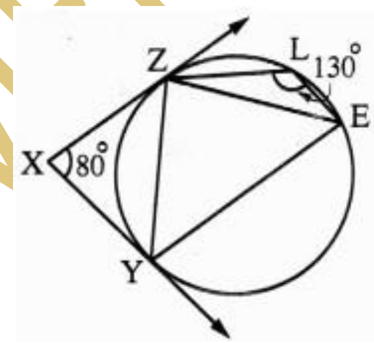
$\overrightarrow{XZ}$ and $\overrightarrow{XY}$ are two tangents at Z and Y,
m (∠ YXZ) = 80° , m (∠ ELZ) = 130°

Prove that:

(1) ZE = ZY

(2) $\overleftrightarrow{XZ} // \overleftrightarrow{YE}$

(3) $\overleftrightarrow{ZE}$ is a tangent to the circle passing through the points X , Y and Z



22) a) $\overline{AB}$ is a diameter in the circle M , $\overline{AC}$ is a chord in it where m (∠ BAC) = 30° , $\overline{BC}$ is drawn and $\overrightarrow{MD}$ is drawn perpendicular to $\overline{AC}$ and intersect it in D ,

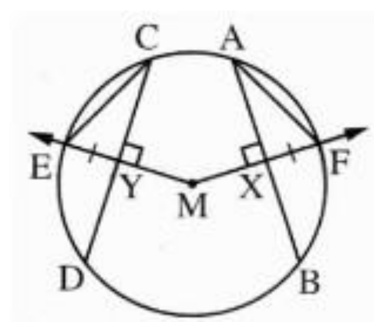
Prove that

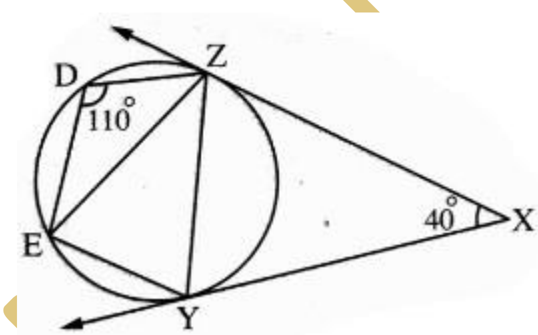
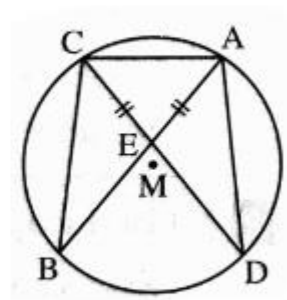
(1) $\overline{MD} // \overline{BC}$ (2) The length of $\overline{BC}$ = length of radius.

b) In the opposite figure:

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M ,
$\overrightarrow{MX} \perp \overline{AB}$ and intersect it at F
, $\overrightarrow{MY} \perp \overline{CD}$ and intersect it at E
, FX = EY

Prove that : (1) AB = CD (2) AF = CE



23)	a) Prove that: In a cyclic quadrilateral each two opposite angles are supplementary.
	b) In the opposite figure: $\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle from point X , m (∠ D) = 110° , m (∠ X) = 40° Prove that m $\overset{\frown}{(ZE)}$ = m $\overset{\frown}{(ZY)}$ 
24)	a) A is a point outside a circle M , $\overrightarrow{AB}$ is a tangent to the circle at point B , $\overrightarrow{AM}$ intersects the circle M at C and D respectively , m (∠ A) = 40° and draw $\overline{BM}$ Find with proof: m (∠ BDC)
	b) In the opposite figure $\overline{AB}$, $\overline{CD}$ are two chords in the circle M intersecting at E , If AE = CE Prove that: m (∠ ACB) = m (∠ CAD) 
25)	$\overline{AB}$ is a diameter in the circle M , $\overline{AC}$ is a chord in this circle and D is the midpoint of $\overline{AC}$, $\overrightarrow{DM}$ was drawn to intersect the tangent to the circle M at B in E Prove that: (1)The figure ADBE is cyclic quadrilateral. (2) m (∠ CMB) = 2 m (∠ MEB)

The answer

1)	70°	2)	parallel
3)	50°	4)	40°
5)	180°	6)	120°
7)	65°	8)	$\frac{1}{2}\pi r$
9)	supplementary	10)	distant
11)	70°	12)	75°
13)	3	14)	rhombus
15)	60°	16)	70°
17)	obtuse	18)	60°
19)	a tangent to the circle	20)	=
21)	4	22)	$\frac{1}{3}$
23)	right	24)	25
25)	360°	26)	90°
27)	bisectors of its interior angles	28)	32°
29)	5	30)	2
31)	70°	32)	50°
33)	40°	34)	80°
35)	equal in length	36)	50 cm

1)(a) ∵ X is midpoint of $\overline{AB}$ ∴ $\overline{MX} \perp \overline{AB}$

∵ Y is midpoint of $\overline{AC}$ ∴ $\overline{MY} \perp \overline{AC}$

∵ The sum of measure of the interior angle of the quadrilateral AXMY = 360°

∴ m ($\angle XMY$) = 360° − (70° + 90° + 90°) = 110°

∴ m ($\angle DME$) = 110° (Q.E.D. 1)

∴ AB = AC ∴ MX = MY

∴ MD = ME (lengths of two radii)

By subtracting

∴ XD = YE (Q.E.D. 2)

(b) ∵ $\overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord

∴ $\overline{AB} \perp \overline{MN}$ ∴ m ($\angle AEN$) = 90°

∵ The sum of measure of the interior angle of the quadrilateral CDNE = 360°

∴ m ($\angle CDN$) = 360° − (50° + 130° + 90°) = 90°

∴ $\overleftrightarrow{ND} \perp \overleftrightarrow{CD}$

∴ $\overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

2)(a) ∵ $\overleftrightarrow{DC}$ // $\overline{AB}$

∴ m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BC}$)

, ∴ $\overline{AB}$ is a diameter in circle M

∴ m ($\overset{\frown}{ACB}$) = 180°

∴ m ($\overset{\frown}{AC}$) = 180° ÷ 2 = 90°

, ∴ m ($\angle DCA$) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$)

∴ m ($\angle DCA$) = $\frac{1}{2}$ × 90° = 45° (The req.)

(b) ∵ $\overline{DE}$ // $\overline{BC}$ ∴ m ($\overset{\frown}{DB}$) = m ($\overset{\frown}{EC}$)

∴ m ($\angle BAD$) = m ($\angle EAC$)

adding m ($\angle BAC$) to both sides

∴ m ($\angle DAC$) = m ($\angle BAE$) (Q.E.D.)

3)(a) ∵ m (∠ EDC) = m (∠ EBC)

(two inscribed angle subtended by $\overset{\frown}{EC}$)

∴ m (∠ EBC) = 30°

, ∵ $\overline{AD}$ // $\overline{BE}$, $\overline{AB}$ is a transversal

∴ m (∠ A) + m (∠ ABE) = 180°

(two interior angle on the same side of the transversal)

∴ m (∠ ABE) = 180° − 100° = 80°

∴ m (∠ ABC) = 80° + 30° = 110°

, ∵ ABCD is a cyclic quadrilateral

∴ m (∠ ABC) + m (∠ ADC) = 180°

∴ m (∠ ADC) = 180° − 110° = 70° (the req.)

(b) ∵ m (∠ BCD) = $\frac{1}{2}$ m ($\overset{\frown}{BMD}$)

(inscribed and central angle subtended by $\overset{\frown}{AD}$)

∵ m (∠ BCD) = $\frac{1}{2}$ × 140° = 70°

, ∵ $\overline{AB}$ // $\overline{CD}$, $\overline{BC}$ is a transversal

∴ m (∠ ABC) = m (∠ BCD) = 70° (alternate angle)

, ∵ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = 70°

∴ in Δ ABC :

m (∠ A) = 180° − (2 × 70°) = 40° (the red.)

4)(a) $\because \overline{AB}$ and $\overline{AC}$ are two tangents to the smaller circle

$\therefore \overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

$\therefore m(\angle MDA) = m(\angle MEA) = 90^\circ$

$\therefore$ From the quadrilateral ADME :

$m(\angle DME) = 360^\circ - (90^\circ + 70^\circ + 90^\circ) = 110^\circ$ (First req.)

$\because$ MD = ME (two radii in the smaller circle)

$\therefore$ AB = AC (second req.)

(b) $\because$ D is the midpoint of the chord $\overline{EC}$

$\therefore \overline{MD} \perp \overline{EC}$ $\therefore m(\angle MDC) = 90^\circ$

$\because \overleftrightarrow{BC}$ is a tangent to the circle at C

$\therefore \overline{MC} \perp \overleftrightarrow{BC}$ $\therefore m(\angle MCB) = 90^\circ$

$\because \overline{AB} // \overline{MC}$, $\overleftrightarrow{BC}$ is a transversal to them

$\therefore m(\angle MCB) + m(\angle ABC) = 180^\circ$

(two interior angle in the same side of the transversal)

$\therefore m(\angle ABC) = 180^\circ - 90^\circ = 90^\circ$

$\therefore m(\angle ADC) + m(\angle ABC) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

5)(a) $\overline{MD} \perp \overline{AB}$ $\therefore m(\angle ADM) = 90^\circ$

, $\because \overline{AC}$ is a diameter in the circle M

$\therefore m(\angle ABC) = 90^\circ$

$\because m(\angle ADM) = m(\angle ABC) = 90^\circ$

and they are corresponding angles.

$\therefore \overline{MD} // \overline{BC}$ (First req.)

In ΔABC : $\because m(\angle A) = 30^\circ$, $m(\angle ABC) = 90^\circ$

$\therefore m(\angle C) = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$ (second req.)

(b) $\because \overline{MD} \perp \overline{AB}$ $\therefore$ D is the midpoint of $\overline{AB}$

$\because \overline{ME} \perp \overline{AC}$ $\therefore$ E is the midpoint of $\overline{AC}$

$\because$ MD = ME $\therefore$ AB = AC (1)

From the quadrilateral ADME

$m(\angle A) = 360^\circ - (120^\circ + 90^\circ + 90^\circ) = 60^\circ$ (2)

From (1) and (2) :

$\therefore \Delta ABC$ is an equilateral triangle. (Q.E.D.)

6)(a) In Δ ABC : $\because$ AB = AD

$\therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 30^\circ = 120^\circ$

, $\because m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 120^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

(b) $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle DCA) = m(\angle DBA)$ (1)

(drawn on $\overline{AD}$ and on the same side of it

, $\because \overline{BD}$ bisects $\angle$ ABC

$\therefore m(\angle DBC) + m(\angle DBA)$ (2)

From (1) , (2) : $\therefore m(\angle DBC) + m(\angle DCA)$

$\therefore \overrightarrow{CD}$ is a tangent to the circle passing through the vertices of ΔBEC (Q.E.D.)

7)(a) $\because$ m ($\angle CMD$) = 2 m ($\angle CAD$)

(central and inscribed angles subtended by $\overset{\frown}{CD}$)

$\therefore$ m ($\angle CMD$) = 2 × 35° = 70°

, $\because \overline{AB} \,//\, \overline{DM}$, $\overleftrightarrow{BM}$ is a transversal

$\therefore$ m ($\angle ABC$) = m ($\angle CMD$)(corresponding angles)

$\therefore$ m ($\angle ABC$) = 70° (The red.)

(b) $\because$ m ($\angle ABC$) = $\frac{1}{2}$ [m ($\overset{\frown}{AC}$) − m ($\overset{\frown}{DE}$)]

$\therefore$ m ($\angle ABC$) = $\frac{1}{2}$ [120° − 50°]

$= \frac{1}{2} \times 70° = 35°$

9)(a) $\because \overleftrightarrow{MN}$ is the line of centers , $\overline{AB}$ is the common chord

$\therefore \overline{AB} \perp \overline{MN}$ $\therefore$ m ($\angle AEN$) = 90°

$\because$The sum of the measures of the interior angles of the quadrilateral CDNE=360°

$\therefore$m ($\angle$ CDN) =360°-- (55°+125°+90°) =90°

$\therefore \overline{ND} \perp \overleftrightarrow{CD}$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\because$ MF = ME (lengths of two radii)

, XF = YE $\therefore$ MX = MY

$\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$\therefore$AB = CD (Q.E.D.1)

$\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$ $\therefore$ AX = $\frac{1}{2}$AB

$\because \overline{MY} \perp \overline{CD}$

$\therefore$Y is the midpoint of $\overline{CD}$ $\therefore$ CY = $\frac{1}{2}$ CD

$\because$ AB = CD $\therefore$ AX = CY

$\therefore \Delta$ AXF, Δ CYE

8)(a)In the small circle

$\because$ m ($\angle XAB$)(the tangency angle)

= m ($\angle ADB$)(the inscribed angle) (1)

In the great circle

$\because$ m ($\angle XAC$)(the tangency angle)

= m ($\angle AEC$)(the inscribed angle) (2)

From (1) and (2) :

$\therefore$ m ($\angle ADB$) = m ($\angle AEC$) but they are corresponding

$\therefore \overline{DB} \,//\, \overline{EC}$ (Q.E.D.)

(b) $\because \overrightarrow{xy}$ and $\overrightarrow{xz}$ are two tangents

$\therefore$ XY = XZ

$\therefore$ In Δ XYZ : m ($\angle XZY$) = m ($\angle XYZ$)

$= \frac{180° - 40°}{2} = 70°$

, $\because$ m ($\angle XZY$) (tangency) = m ($\angle$ YEZ) (inscribed)

$\therefore$ m ($\angle$ YEZ) = 70°

, $\because$ DEYZ is a cyclic quadrilateral

$\therefore$ m ($\angle$ EYZ) + m ($\angle$ D) = 180°

$\therefore$ m ($\angle$ EYZ) = 180° − 110° = 70°

$\therefore$ m ($\angle$ EYZ) = m ($\angle$ YEZ) = 70°

$\therefore$ In Δ EZY : ZE = ZY

$\therefore$ m ($\overset{\frown}{ZDE}$) = m ($\overset{\frown}{ZY}$)

In them $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$

$\therefore \Delta AXF \equiv \Delta CYE$ then we deduce that $AF = CE$

(Q. E. D. 2)

10) $\because \overline{DE} // \overline{BC}$ $\therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{CE})$

$\therefore m(\angle DAB) = m(\angle CAE)$

adding $m(\angle BAC)$ to both sides

$\therefore (\angle DAC) = m(\angle BAE)$ (Q.E.D.)

(b) $\because \overline{LE} // \overrightarrow{XN}$, $\overleftrightarrow{XZ}$ is a transversal

$\because m(\angle XEL) = m(\angle NXZ)$ (alternate angles)

$\because m(\angle y)$ the inscribed $= m(\angle NXZ)$ of tangency

$\therefore m(\angle y) = m(\angle XEL)$

$\therefore$ the figure LYZE is a cyclic quadrilateral.

(Q.E.D.)

11)(a) $\because \overline{AB}$ touches the circle at B $\therefore \overline{MB} \perp \overline{AB}$

$\because \overline{AC}$ touches the circle at C $\therefore \overline{MC} \perp \overline{AC}$

$\therefore (\angle ABM) + m(\angle ACM) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ the figure ABMC is a cyclic quadrilateral.

(Q.E.D. 1)

$\because \angle CMD$ is an exterior angle of it

$\therefore m(\angle CMD) = m(\angle A) = 45^\circ$

In ΔMCD: $M(\angle D) = 180^\circ - (90^\circ + 45^\circ) = 45^\circ$

$\therefore CD = MC$ (1)

$\because \overline{AC}, \overline{AB}$ are two tangent segments to the circle

$\therefore AC = AB$ (2)

Adding (1) and (2) : $\therefore CD + AC = MC + AB$

$\therefore AD = AB + MC$,

$\because MC = MB$ (the length of two radii)

$\therefore AD = AB + MB$ (Q. E.D.2)

(b) In ΔCBD : $\because CB = CD$

$\therefore m(\angle CBD) = m(\angle CDB) = 40^\circ$

$\therefore m(\angle C) = 180^\circ - 2 \times 40^\circ = 100^\circ$

, $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle A) + m(\angle C) = 180^\circ$

$\therefore m(\angle A) = 180^\circ - 100^\circ = 80^\circ$ (The req.)

12)(a) In the small circle

$\because m(\angle XAB)$ (the tangency angle)

$= m(\angle ADB)$ (the inscribed angle) (1)

In the great circle

$\because m(\angle XAC)$ (the tangency angle)

$= m(\angle AEC)$ (the inscribed angle) (2)

From (1) and (2) :

$\therefore m(\angle ADB) = m(\angle AEC)$ but they are corresponding.

$\therefore \overline{DB} // \overline{EC}$ (Q.E.D.)

(b) $\because AB = CD$ $\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$

Subtracting $m(\overset{\frown}{BD})$ from both sides

$\therefore m(\overset{\frown}{AD}) = m(\overset{\frown}{BC})$ $\therefore m(\angle C) = m(\angle A)$

$\therefore \Delta ACE$ is isosceles (Q.E.D.)

13)(a) $\because \overleftrightarrow{MN}$ is the line of centers, $\overline{AB}$ is the common chord

$\therefore \overline{AB} \perp \overleftrightarrow{MN}$ $\quad\quad \therefore m(\angle AEN) = 90^\circ$

$\because$The sum of the measures of the interior angles of the quadrilateral CDNE=360°

$\therefore m(\angle CDN) = 360^\circ - (55^\circ + 125^\circ + 90^\circ) = 90^\circ$

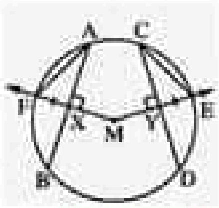
$\therefore \overleftrightarrow{ND} \perp \overleftrightarrow{CD}$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\because MF = ME$ (lengths of two radii)

$, XF = YE$

$\therefore MX = MY$



$\because \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{CD}$

$\therefore AB = CD$ (Q.E.D.1)

$\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$ $\quad \therefore AX = \frac{1}{2}AB$

$\because \overline{MY} \perp \overline{CD}$

$\therefore$Y is the midpoint of $\overline{CD}$ $\quad \therefore CY = \frac{1}{2} CD$

$\because AB = CD$

$\therefore AX = CY$

$\therefore \Delta AXF, \Delta CYE$

In them $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$

$\therefore \Delta AXF \equiv \Delta CYE$ then we deduce that $AF = CE$

(Q. E. D. 2)

14)(a) $\because m(\angle E) = \frac{1}{2}[m(\overset{\frown}{BD}) - m(\overset{\frown}{AO})]$

$\therefore m(\angle E) = \frac{1}{2} m(\overset{\frown}{BD}) - \frac{1}{2}(\overset{\frown}{AO})$

$, \because m(\angle DCB) = \frac{1}{2} m(\overset{\frown}{BD})$

$\therefore m(\angle E) = m(\angle DCB) - \frac{1}{2} m(\overset{\frown}{AO})$

$\therefore m(\angle DCB) = m(\angle E) + \frac{1}{2} m(\overset{\frown}{AO})$

$\therefore m(\angle DCB) > m(\angle E)$ (Q. E. D.)

(b) $\because \angle ABE$ is an exterior angle of the cyclic quadrilateral ABCD

$\therefore m(\angle d) = m(\angle ABE) = 100^\circ$

In Δ ACD : $m(\angle ACD)$

$= 180^\circ - (100^\circ + 40^\circ) = 40^\circ$

$\therefore m(\angle ACD) = m(\angle CAD)$

$\therefore CD = AD$ $\quad \therefore m(\overset{\frown}{CD}) = m(\overset{\frown}{AD})$ (Q.E.D.)

15)(a) an axis of symmetry.

(b) $\because \overline{DE} \parallel \overline{BC}$ $\quad \therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{CE})$

$\therefore m(\angle DAB) = m(\angle CAE)$

adding $m(\angle BAC)$ to both sides

$\therefore (\angle DAC) = m(\angle BAE)$ (Q. E. D.)

16)(a)

$\because \overline{LE} \parallel \overrightarrow{XN}, \overleftrightarrow{XZ}$ is a transversal

$\because m(\angle XEL) = m(\angle NXZ)$ (alternate angles)

$\because m(\angle y)$ the inscribed $= m(\angle NXZ)$of tangency

$\therefore m(\angle y) = m(\angle XEL)$

$\therefore$ the figure LYZE is a cyclic quadrilateral

(Q. E. D.)

17)(a)

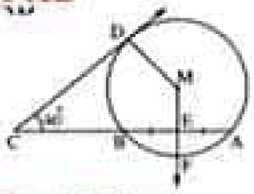
$\because \overline{CD}$ is a tangent to the circle

$\therefore \overline{MD} \perp \overline{CD}$ $\quad \therefore (\angle MDC) = 90^\circ$

$, \because$ E is the midpoint of $\overset{\frown}{AB}$

$\therefore \overline{ME} \perp \overline{AB}$

$\therefore m(\angle MEC) = 90^\circ$



$\therefore m(\angle DMF)$

$= 360^\circ - (40^\circ + 90^\circ + 90^\circ)$

$= 360^\circ - 220^\circ - 140^\circ$

$, \because AE = \frac{1}{2} AB = 8$ cm

$, \because AM = r = 10$ cm

In Δ AEM : $\because m(\angle AEM) = 90^\circ$

$\therefore (ME)^2 = (AM)^2 - (AE)^2 = 100 - 64 = 36$

$\therefore ME = \sqrt{36} = 6$ cm

$\therefore FE = MF - ME = 10 - 6$cm. (second req.)

(b) $\therefore \overline{MN}$ is the line of centres

$, \overline{AB}$ is the common chord of the two circles

$\therefore \overline{MN} \perp \overline{AB}$

$, \because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AC}$

$, \because AB = AC$ $\quad \therefore MX = MD$

$\because MY = ME$ (lengths of two radii)

$\therefore$ MY-MX=ME-MD

$\therefore$ XY=DE (Q.E.D)

---

18)

$\because \overline{AB}$ is a diameter of the circle M

$\therefore m(\angle ADB) = 90^\circ$

In Δ ABD: $\because m(\angle ABD) = 40^\circ$

$\therefore m(\angle A) = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$

$, \because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle C) + m(\angle A) = 180^\circ$

$\therefore m(\angle C) = 180^\circ - 50^\circ = 130^\circ$ (The req)

---

19) $\because \overline{DE} // \overline{BC}$

$\therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{CE})$

adding m ($\angle$ BAC)to both sides

$\therefore m(\angle DAC) = m(\angle BAE)$ (Q. E. D)

---

20)(a)

$\because m(\angle DMC) = 2\, m(\angle CAD)$

(central and inscribed angles subtended by $\overset{\frown}{CD}$)

$\therefore m(\angle DMC) = 2 \times 30^\circ = 60^\circ$

$, \because \overline{AB} // \overline{DM}. \overline{BC}$ is a transbersal

$, \because m(\angle B) = m(\angle DMC)$

$= 60^\circ$ (corresponding angles)

$, \because \overline{BC}$ is a diameter of circle M

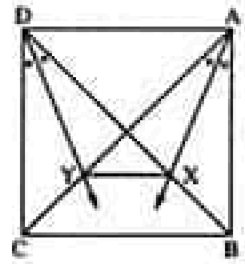
$\therefore m(\angle BAC) = 90^\circ$

$\therefore$ In Δ ABC: $m(\angle ACB) = 180^\circ - (90^\circ + 60^\circ)$

$= 30^\circ$ (The req)

(b)

$\because$ ABCD is a square. $\overline{AC}$ and $\overline{BD}$ are two diagonals of the square



$\therefore m(\angle BAC) = m(\angle BDC)$

$\therefore \frac{1}{2} m(\angle BAC)$

$= \frac{1}{2} m(\angle BDC)$

$\therefore m(\angle XAY)$

$= m(\angle XDY)$ but they are drawn On $\overline{XY}$ and on one side of it

$\therefore$ The figure AXYD is a cyclic quadrilateral

(Q. E. D. 1)

$\because m(\angle DXY) = m(\angle DAY) = 45^\circ$

(They are drawn on $\overline{DY}$ and on one side of it)

(Q. E. D. 2)

21)(a)

∵ $\overline{XY}, \overline{XZ}$ are tangentsegments to the circle at Y and Z

∴ XY = XZ

$\therefore m(\angle XYZ) = m(\angle XZY) = \frac{180^\circ - 80^\circ}{2} = 50^\circ$

∴ m (∠ ZEY) (the inscribed angle)

= m (∠ ZYX) (the tangency angle) = 50°

∵ The figure LEYZ. is a cyclic quadrilateral

∴ $m(\angle ZYE) = 180^\circ - 130^\circ = 50^\circ$

∴ $m(\angle ZEY) = m(\angle ZYE) = 50^\circ$

∴ ZE = ZY (Q. E. D. 1)

∵ $m(\angle XZY) = m(\angle ZYE) = 50^0$

but they are alternate angles

∴ $\overline{XZ} // \overline{YE}$ (Q. E. D. 2)

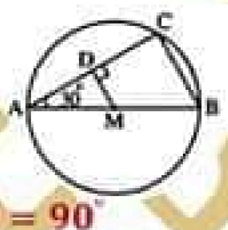
∵ In Δ ZYE: $m(\angle EZY) = 180^\circ - 2 \times 50^\circ = 80^\circ$

∴ $m(\angle EZY) = m(\angle X) = 80^\circ$

∴ $\overline{ZE}$ is a tangent to the circle passing through The points X, Y and Z (Q. E. D. 3)

---

22)(a)



∵ $\overline{AB}$ is a diameter

∴ $m(\angle C) = 90^\circ$.

∵ $\overline{MD} \perp \overline{AC}$

∴ $m(\angle ADM) = m(\angle C) = 90^\circ$

and they are corresponding angles

∴ $\overline{DM} // \overline{BC}$ (Q. E. D. 1)

In Δ ABC which is right − anlged at C

∴ $m(\angle A) = 30^\circ$

∴ $BC = \frac{1}{2} AB$

= radius length (Q. E. D. 2)

(b) ∵ MF = ME (Two radii) (1)

, XF = YE (given) (2)

Subtracting (2) from (1) :

∴ MX = MY, ∵ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

∴ AB = CD (Q.E.D.1)

$\therefore \frac{1}{2} AB = \frac{1}{2} CD$ ∴ $AX = CY$ (Q.E.D.1)

∴ In ΔΔ AXF , CYE :

$\begin{cases} AX = CY \\ XF = YE \\ m(\angle CYE) = m(\angle AXF) = 90^\circ \end{cases}$

∴ Δ AXF ≡ Δ CYE ∴ AF = CE (Q. E. D. 2)

---

23)(a) Theoretical.

(b) ∵ (XZ) and (XY) are two tangents ∴ XZ=XY

$\therefore m(\angle XZY) = \frac{(180^\circ - 40^\circ)}{2} = 70^\circ$

∴ m (∠ ZEY) the inscribed

= m (∠ XZY) of tangency

∴ $m(\angle ZEY) = 70^\circ$ (1)

,∵ DEYZ is a cyclic quadrilateral

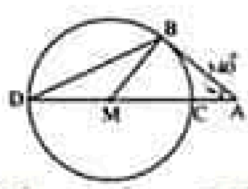
∴ $m(\angle EYZ) = 180^\circ - 110^\circ = 70^\circ$ (2)

From (1) and (2):

∴ $m(\overset{\frown}{ZE}) = m(\overset{\frown}{ZY})$ (Two arcs subtended by two equal inscribed angles in measure)

---

24)(a) ∵ $\overline{AB}$ is tangent to the circle M at B



∴ $\overline{MB} \perp \overline{AB}$

∴ From Δ ABM:

$m(\angle BMA) = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$

$\therefore m(\angle D) = \frac{1}{2} m(\angle BMC) = 25^\circ$

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

(b) In Δ AEC: ∵ EA = EC

∴ m (∠ BAC) = m(∠DCA) (1)

, ∵ m(∠BAD) = m (∠DCB)

(inscribed angles subtended by $\overset{\frown}{BD}$) (2)

,adding (1) and (2)

∴ m (∠ CAD) = m(∠ ACB) (Q. E. D)

25)(a)

$\because$ D is the midpoint of the chord $\overline{AC}$

$\therefore$ $\overline{MD} \perp \overline{AC}$.

$\because$ $\overline{BE}$ is tangent to the circle M at B

$\therefore$ $\overline{MB} \perp \overline{BE}$

$\therefore$ $m(\angle ADE) = m(\angle EBA) = 90^\circ$

and they aer draw on $\overline{AE}$ and on one side of it

$\therefore$ ADBE is a cyclic quadrilateral (Q. E. D. (1))

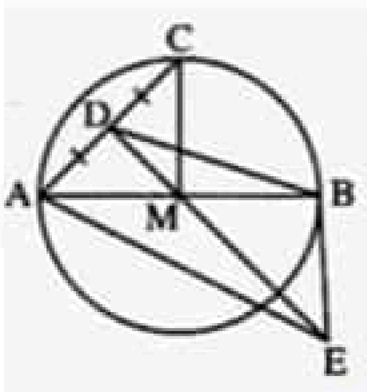
$\therefore$ $m(\angle BED) = m(\angle BAD)$

(are drawn on $\overline{BD}$ and on one side of it)

, $\because$ $m(\angle CMB) = 2\, m(\angle CAB)$

(central and inscribed angles subtended by $\overset{\frown}{BC}$)

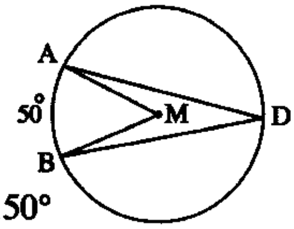
$\therefore$ $m(\angle CMB) = 2m(\angle MEB)$ (Q. E. D. 2)



SECOND: GEOMETRY

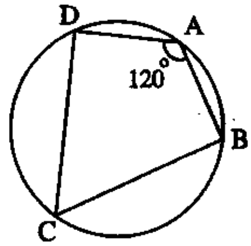
Choose the correct answer :

1.	The inscribed angle drawn in a semicircle is .... (a) an acute. (b) an obtuse. (c) a straight. (d) a right.
2.	**In the opposite figure :** Circle of centre M If m $(\overset{\frown}{AB}) = 50°$, then m (∠ ADB) = ... (a) 25° (b) 50° (c) 100° (d) 150°
3.	The number of symmetric axes of any circle is .... (a) zero (b) 1 (c) 2 (d) an infinite number.



4. **In the opposite figure :**

If m (∠ A) = 120° , then m (∠ C) = ...



(a) 60°   (b) 90°

(c) 120°   (d) 180°

5. If the straight line L is a tangent to the circle M of diameter length 8 cm. , then the distance between L and the centre of the circle equals .... cm.

(a) 3   (b) 4   (c) 6   (d) 8

6. The surface of the circle M ∩ the surface of the circle N = {A} and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm.

(a) 5   (b) 6   (c) 11   (d) 16

7. The measure of the arc which equals half the measure of the circle equals ....

(a) 360°   (b) 180°   (c) 120°   (d) 90°

8. The number of common tangents of two touching circles externally equals ....

(a) 0   (b) 1   (c) 2   (d) 3

9. The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45°   (b) 90°   (c) 120°   (d) 80°

10. The angle of tangency is included between ....

(a) two chords.   (b) two tangents.

(c) a chord and a tangent.   (d) a chord and a diameter.

11. ABCD is a cyclic quadrilateral , m (∠ A) = 60° , then m (∠ C) = ...

(a) 60°   (b) 30°   (c) 90°   (d) 120°

12. If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm. , then MN = ... cm.

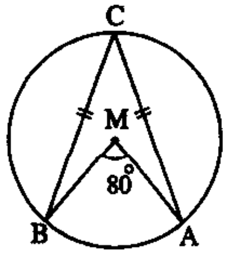
(a) 14   (b) 4   (c) 5   (d) 9

13. **In the opposite figure :**

m (∠ ACB) = ...

(a) 40° (b) 80°

(c) 90° (d) 180°



14. The number of the common tangents of two distant circles is ....

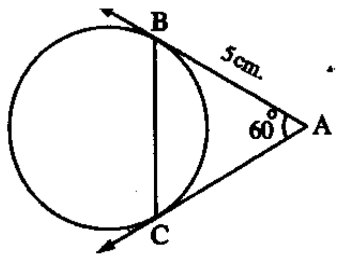
(a) 1 (b) 2 (c) 3 (d) 4

15. **In the opposite figure :**

The length of $\overline{BC}$ = ... cm.

(a) 3 (b) 4

(c) 5 (d) 6



16. The number of circles which can be drawn passes through the endpoints of a line segment $\overline{AB}$ equals ....

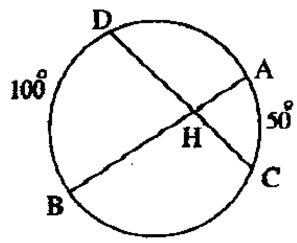
(a) 1 (b) 2 (c) 3 (d) an infinite number.

17. **In the opposite figure :**

m (∠ AHC) = ...

(a) 25° (b) 50°

(c) 75° (d) 100°



18. The measure of the inscribed angle is .... the measure of the central angle , subtended by the same arc.

(a) half (b) third (c) quarter (d) double

19. It is possible to draw a circle passing through the vertices of a ....

(a) trapezium. (b) parallelogram. (c) rectangle. (d) rhombus.

20. The centre of the inscribed circle of any triangle is the point of intersection of its ....

(a) altitudes. (b) medians.

(c) axes of symmetry of its sides. (d) bisectors of its interior angles.

21. If the two circles M and N are touching internally , the radius length of one of them = 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm.

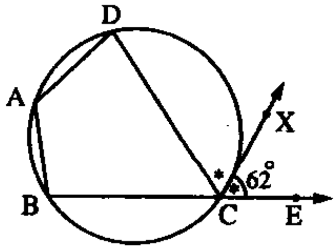
(a) 12 (b) 11 (c) 6 (d) 5

22. **In the opposite figure :**

If $E \in \overrightarrow{BC}$, $\overrightarrow{CX}$ bisects $\angle$ DCE

, m ($\angle$ XCE) = 62°

, then m ($\angle$ A) = ...



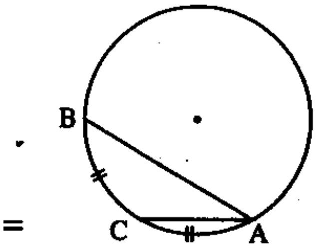
(a) 62° (b) 118° (c) 56° (d) 124°

---

23. **In the opposite figure :**

If C is the midpoint of $\overset{\frown}{AB}$

, then AB .... 2 AC



(a) < (b) > (c) ≥ (d) =

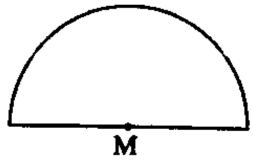
---

24. The two opposite angles in the cyclic quadrilateral are ....

(a) equal. (b) supplementary. (c) complementary. (d) alternate.

---

25. The opposite figure represents a semicircle its centre is M and its radius length is r length unit, then the area of the opposite figure = ... square units.



(a) $2\pi r$ (b) πr (c) πr^2 (d) $\frac{\pi r^2}{2}$

---

26. In a regular hexagon , the measure of the angle of its vertex equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

---

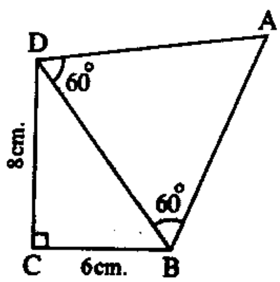
27. If $\overline{AB}$ is a line segment , then the number of circles can be drawn passing through A and B equals ....

(a) 1 (b) 2 (c) 3 (d) an infinite number.

---

28. **In the opposite figure :**

The length of $\overline{AB}$ = ... cm.



(a) $10\sqrt{3}$ (b) 10

(c) 5 (d) $5\sqrt{3}$

---

29. The inscribed angle which is opposite to the minor arc in a circle is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

30. If the area of the circle is 9π cm^2 , then its radius length = ... cm.

(a) 9 (b) 2 (c) (– 3) (d) 3

31. The number of symmetric axes of a square = ...

(a) 1 (b) 2 (c) 3 (d) 4

32. If M is a circle of a diameter length equals 14 cm. , MA = $(2x + 3)$ cm. where A lies on the circle , then x = ...

(a) 5 (b) 3 (c) 2 (d) 1

33. The raito between the measure of the inscribed angle and the measure of the central angle subtended by the same arc = ...

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

34. If ABCD is a cyclic quadrilateral and m (∠ B) = $\frac{1}{2}$ m (∠ D) , then m (∠ B) = ...

(a) 90° (b) 60° (c) 120° (d) 180°

35. If the figure ABCD ~ the figure XYZL , then m (∠ B) = m (∠ ....)

(a) X (b) Y (c) Z (d) L

36. The two tangents which are drawn from the two endpoints of a diameter of a circle are ....

(a) parallel. (b) perpendicular. (c) coincide. (d) intersecting.

37. The number of the axes of symmetry of the semicircle .... the number of the axes of symmetry of the isosceles triangle.

(a) > (b) < (c) = (d) ≥

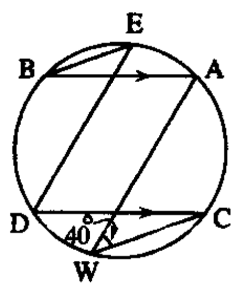
38. **In the opposite figure :**

$\overline{AB}$ // $\overline{CD}$, m (∠ AWC) = 40° ,

then m (∠ DEB) = ...

(a) 50° (b) 40°

(c) 30° (d) 45°



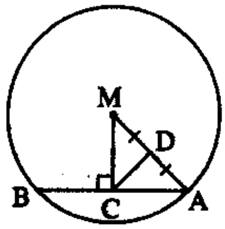
39. **In the opposite figure :**

$CD = 3$ cm. , $\overline{MC} \perp \overline{AB}$

, D is the midpoint of $\overline{MA}$

then the area of the circle M = ... π cm^2.

(a) 3 (b) 6 (c) 9 (d) 36



Essay problems:

1. **Complete and prove that :**

In a cyclic quadrilateral , each two opposite angles are ....

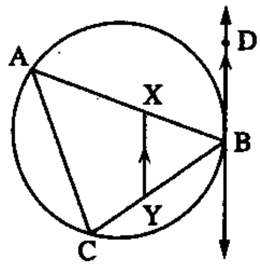
2. **In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, $X \in \overline{AB}$, $Y \in \overline{BC}$ where $\overline{XY} \parallel \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



3. **In the opposite figure :**

Two circles are touching internally at B

, $\overrightarrow{AB}$ is a common tangent

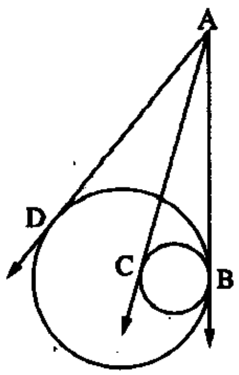
, $\overrightarrow{AC}$ is a tangent to the smaller circle at C

, $\overrightarrow{AD}$ is a tangent to the greater circle at D

, $AC = 15$ cm. , $AB = (2x - 3)$ cm.

and $AD = (y - 2)$ cm.

Find : The value of each of x and y



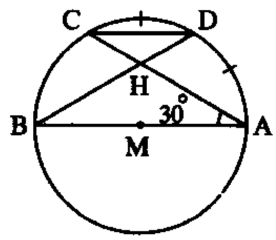
4. **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, C $\in$ the circle M , m ($\angle$ CAB) = 30°

, D is midpoint of $\overset{\frown}{AC}$, $\overline{DB} \cap \overline{AC} = \{H\}$

(1) **Find :** m ($\angle$ BDC) and m ($\overset{\frown}{AD}$) (2) **Prove that :** $\overline{AB} \parallel \overline{DC}$



5. State two cases of a cyclic quadrilateral.

6. **In the opposite figure :**

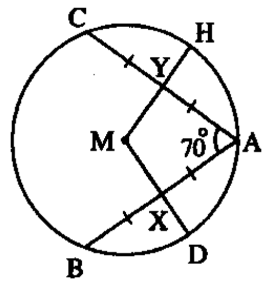
$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in circle M

, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$

, m (∠ CAB) = 70°

(1) **Calculate :** m (∠ DMH)

(2) **Prove that :** XD = YH



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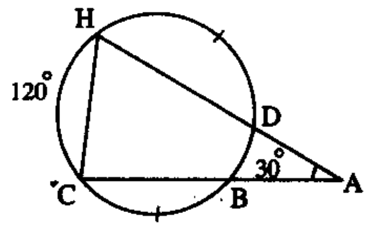
7. **In the opposite figure :**

m (∠ A) = 30° , m ($\overset{\frown}{HC}$) = 120°

, m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DH}$)

(1) **Find :** m ($\overset{\frown}{BD}$ the minor)

(2) **Prove that :** AB = AD



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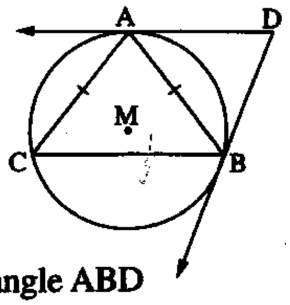
8. **In the opposite figure :**

$\overrightarrow{DA}$ and $\overrightarrow{DB}$ are two tangents of the circle M

and AB = AC

Prove that :

$\overline{AC}$ is a tangent to the circle passing through the vertices of the triangle ABD



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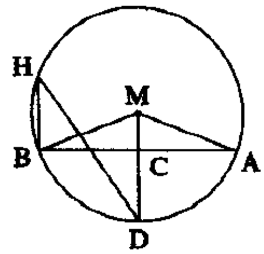
9. **In the opposite figure :**

C is the midpoint of $\overline{AB}$, $\overrightarrow{MC}$ ∩ the circle M = {D}

, m (∠ MAB) = 20°

Find : m (∠ BHD) and m ($\overset{\frown}{ADB}$)



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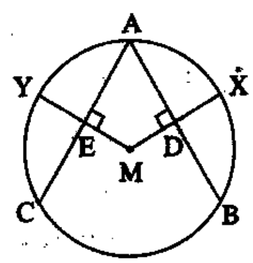
10. **In the opposite figure :**

AB = AC , $\overline{MD} \perp \overline{AB}$,

$\overline{ME} \perp \overline{AC}$

Prove that : XD = YE



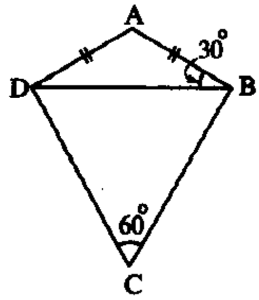
11. **In the opposite figure :**

ABCD is a quadrilateral in which AB = AD ,

m (∠ ABD) = 30° ,

m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



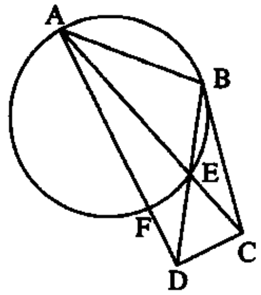
12. **In the opposite figure :**

$\overline{BC}$ is a tangent at B ,

E is the midpoint of $\overset{\frown}{BF}$

Prove that :

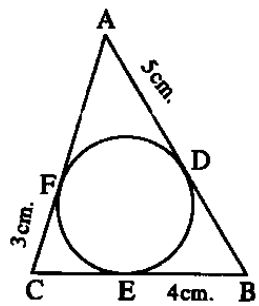
ABCD is a cyclic quadrilateral.



13. **In the opposite figure :**

A circle is drawn touches the sides of a triangle ABC , $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F , AD = 5 cm , BE = 4 cm. , CF = 3 cm.

Find the perimeter of Δ ABC

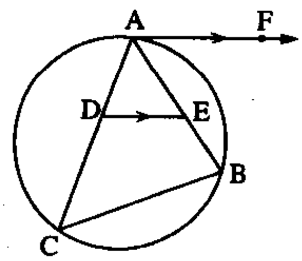


14. **In the opposite figure :**

$\overrightarrow{AF}$ is a tangent to the circle at A , $\overrightarrow{AF}$ // $\overrightarrow{DE}$

Prove that :

DEBC is a cyclic quadrilateral.



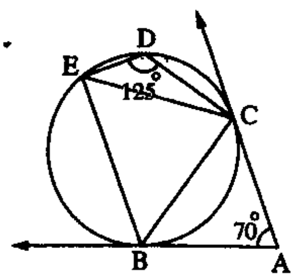
15. **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the Circle at B , C , m (∠ A) = 70° ,

m (∠ CDE) = 125°

Prove that :

(1) CB = CE (2) $\overrightarrow{AC}$ // $\overline{BE}$

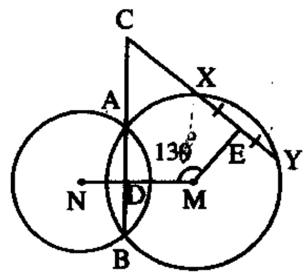


16. **In the opposite figure :**

If E is the midpoint of $\overline{XY}$

, m (∠ EMN) = 130°

, **then find :** m (∠ C)



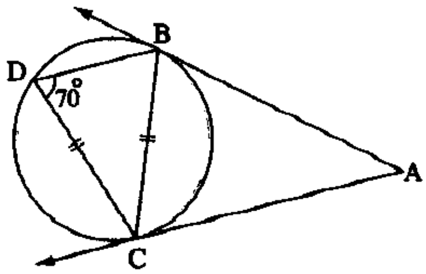
17. **In the opposite figure :**

If $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ D) = 70° , CB = CD

(1) **Find :** m (∠ A)

(2) **Prove that :** $\overline{BD}$ // $\overline{AC}$

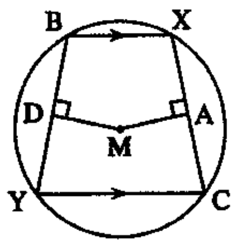


18. **In the opposite figure :**

$\overline{XB}$ // $\overline{CY}$, $\overline{MA}$ ⊥ $\overline{XC}$

, $\overline{MD}$ ⊥ $\overline{BY}$

Prove that : MA = MD



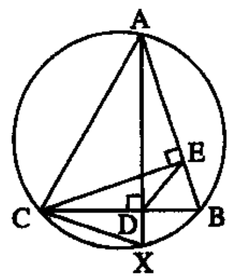
19. **In the opposite figure :**

$\overline{CE}$ ⊥ $\overline{AB}$, $\overrightarrow{AD}$ ⊥ $\overline{BC}$ and intersects the circle at X

Prove that :

(1) AEDC is a cyclic quadrilateral.

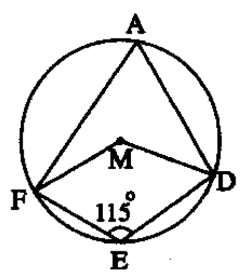
(2) $\overrightarrow{CB}$ bisects ∠ ECX



20. **In the opposite figure :**

If m (∠ DEF) = 115°

, **then find :** m (∠ DMF)



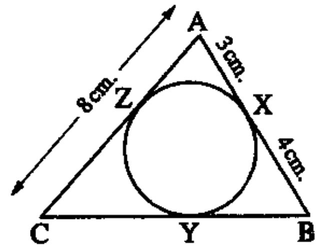
21. **Complete :** The measure of the inscribed angle equals .... the measure of the central angle .... by the same arc.

22. **In the opposite figure :**

Inscribed circle of the triangle ABC touches its sides at X , Y and Z

If AX = 3 cm. , XB = 4 cm. , AC = 8 cm.

Find : The length of $\overline{BC}$

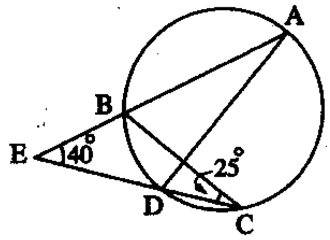


23. **In the opposite figure :**

$\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ C) = 25°

, m (∠ E) = 40°

Find : m (∠ ADC)



24. **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

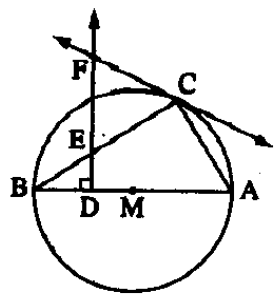
, $\overleftrightarrow{CF}$ is a tangent to the circle at C

, $\overrightarrow{DF} \perp \overline{AB}$ and intersects $\overline{BC}$ at E

Prove that :

(1) ADEC is a cyclic quadrilateral.

(2) Δ FCE is an isosceles triangle.



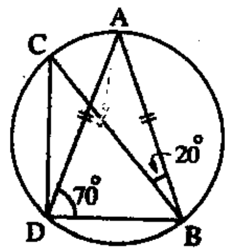
25. **In the opposite figure :**

AB = AD

, m (∠ ABC) = 20°

, m (∠ ADB) = 70°

Find : m (∠ C) , m (∠ BDC)



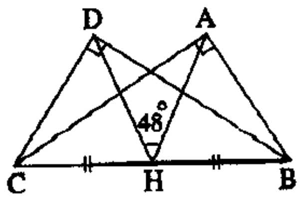
26. **In the opposite figure :**

m (∠ BAC) = m (∠ BDC) = 90°

, H is the midpoint of $\overline{BC}$ and m (∠ AHD) = 48°

(1) **Prove that :** ABCD is a cyclic quadrilateral.

(2) **Find :** m (∠ ABD)



27. Using your geometric tools , draw $\overline{AB}$ with a length of 4 cm. , then draw a circle passing through the two points A and B whose radius length is 3 cm.
What are the possible solutions ? (Don't remove the arcs)

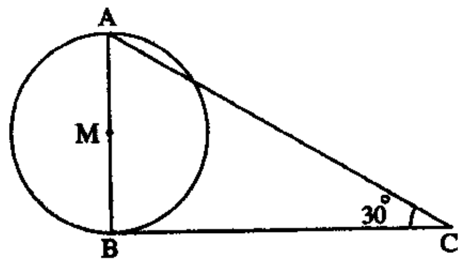
28. **In the opposite figure :**

A circle M of circumference 44 cm.

, $\overline{AB}$ is a diameter , $\overline{BC}$ is a tangent at B

and m (∠ ACB) = 30°

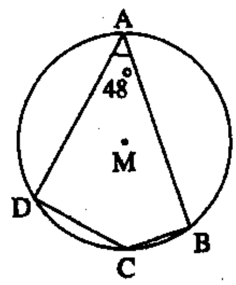
Find : The length of $\overline{BC}$ $\left(\pi = \frac{22}{7}\right)$



29. **In the opposite figure :**

If M is a circle , m (∠ A) = 48°

Find : m ($\overset{\frown}{BD}$ the major)



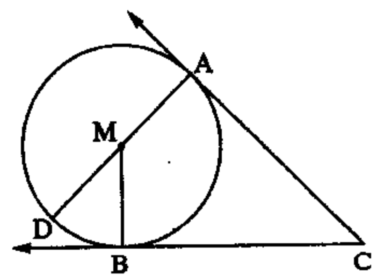
30. **In the opposite figure :**

$\overline{AD}$ is a diameter in a circle M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle M ,

touch it at A and B respectively.

Prove that : m (∠ DMB) = m (∠ ACB)



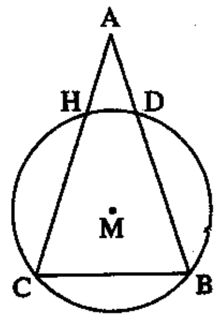
31. **In the opposite figure :**

ABC is a triangle in which AB = AC

, $\overline{BC}$ is a chord in the circle M

, if $\overline{AB}$ and $\overline{AC}$ cut the circle at D and H respectively.

Prove that : m ($\overset{\frown}{DB}$) = m ($\overset{\frown}{HC}$)

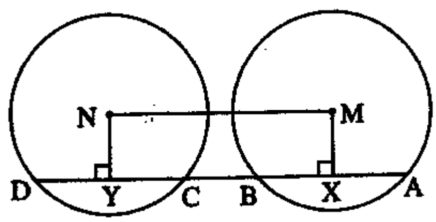


32. **In the opposite figure :**

M and N are two congruent circles

, AB = CD

Prove that : The figure MXYN is a rectangle.

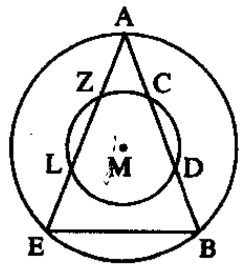


33. ABCD is a quadrilateral inscribed in a circle , H is a point outside the circle and $\overrightarrow{HA}$ and $\overrightarrow{HB}$ are two tangents to the circle at A and B , if m (∠ AHB) = 70° and m (∠ ADC) = 125° , **prove that :**

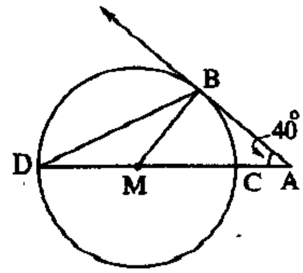
① AB = AC

② $\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and H

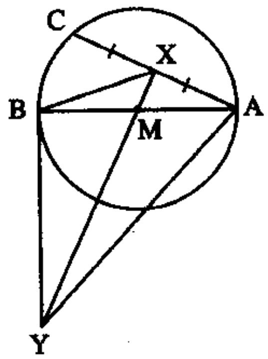
34. **In the opposite figure :**
Two concentric circles at M
, m (∠ ABE) = m (∠ AEB)
Prove that : CD = ZL



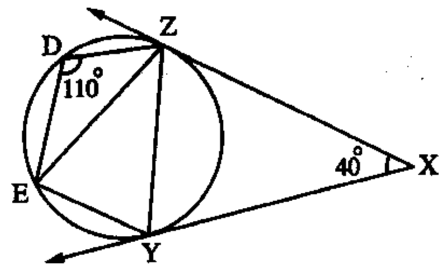
35. **In the opposite figure :**
$\overrightarrow{AB}$ is a tangent to the circle M
, m (∠ A) = 40°
Find with proof : m (∠ BDC)



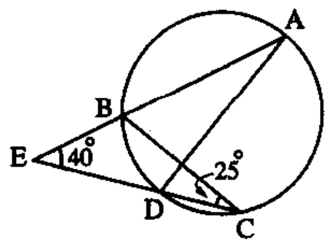
36. **In the opposite figure :**
$\overline{AB}$ is a diameter in the circle M
, X is the midpoint of $\overline{AC}$ and $\overrightarrow{XM}$ intersecting
the tangnet of the circle at B in Y
Prove that : The figure AXBY is a cyclic quadrilateral.



37. **In the opposite figure :**
$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle
at the two points Y and Z , m (∠ X) = 40°
, m (∠ D) = 110°
Prove that : m (∠ ZYE) = m (∠ ZEY)



38. **In the opposite figure :**
m (∠ E) = 40° , m (∠ C) = 25°
Find with proof :
(1) m (∠ ADC)　　　　(2) m ($\overset{\frown}{AC}$)



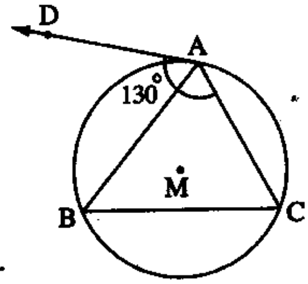
39. ABCD is a quadrilateral drawn in a circle , E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$
, m ($\overset{\frown}{AB}$) = 110° , m (∠ CBE) = 85°
Find with proof : m (∠ BDC)

40. **In the opposite figure :**

$\overrightarrow{AD}$ is the tangent to the circle M at A

, m (∠ DAC) = 130°

Find with proof : m (∠ B)



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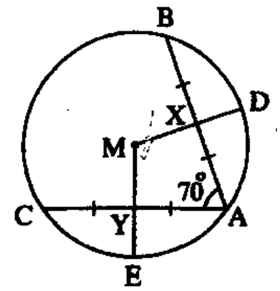
41. **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length at the circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ A) = 70°

(1) **Find :** m (∠ DME) (2) **Prove that :** XD = YE



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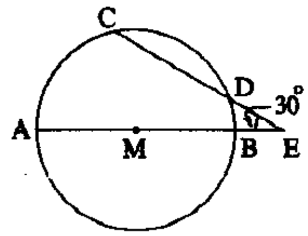
42. **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ E) = 30° , m $(\overset{\frown}{AC})$ = 80°

Find : m $(\overset{\frown}{CD})$



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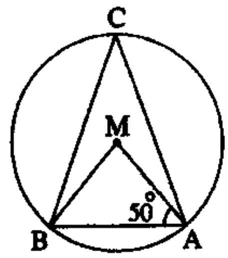
43. **In the opposite figure :**

M is a circle , m (∠ MAB) = 50°

Find : m (∠ C)



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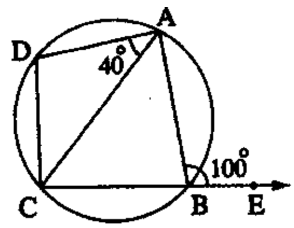
44. **In the opposite figure :**

m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : Δ DAC is an isosceles triangle.



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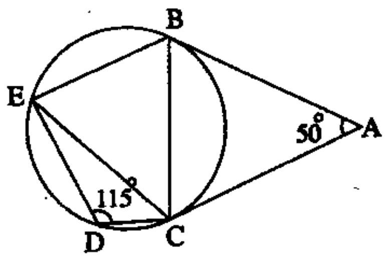
45. **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle at B and C

, m (∠ A) = 50° , m (∠ D) = 115°

Prove that : (1) $\overrightarrow{BC}$ bisects ∠ ABE (2) CB = CE



(33) $2x+y=1 \rightarrow (1)$ $\quad$ ($\times -2$) $\quad x+2y=5 \rightarrow (2)$

$-4x-2y=-2 \rightarrow (3)$

$-3x=3$

$\boxed{x=-1}$ in (1)

$-2+y=1$, $\boxed{y=3}$, S.S. $=\{(-1,3)\}$

(34) $y-x=3$ $\qquad x^2+y^2-xy=13 \rightarrow (2)$

$y=x+3 \rightarrow (1)$ $\qquad$ From (1) in (2)

$x^2+x^2+6x+9-x^2-3x-13=0$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=-4$ or $x=1$

in (1)

$y=-1$ or $y=4$

S.S. $=\{(-4,-1),(1,4)\}$

(35) (1) $0.7+0.6-0.4=0.9$

(2) $0.7-0.4=0.3$

(36) $n(x)=\frac{x(x+1)}{(x-1)(x+1)}-\frac{(x-5)}{(x-5)(x-1)}$

$D=R-\{1,-1,5\}$

$n(x)=\frac{x-1}{x-1}=1$

(37) $a=1$, $b=-2$, $c=-6$

S.S. $=\{3.6, -1.6\}$

(38) $n_1(x)=\frac{x(x+2)}{(x+2)(x+2)}=\frac{x}{x+2}$, $D_1=R-\{-2\}$

$n_2(x)=\frac{2x}{2(x+2)}=\frac{x}{(x+2)}$, $D_2=R-\{-2\}$

$\therefore n_1=n_2$

(39) $D=R-\{-1,2\}$

$n^{-1}(3)=\frac{3+1}{3-2}=\frac{4}{1}=4$

(40) $n_1(x)=\frac{x^2-3x+9}{(x+3)(x^2-3x+9)}=\frac{1}{x+3}$

$D_1=R-\{-3\}$

$n_2(x)=\frac{2}{2(x+3)}=\frac{1}{x+3}$,

$D_2=R-\{-3\}$

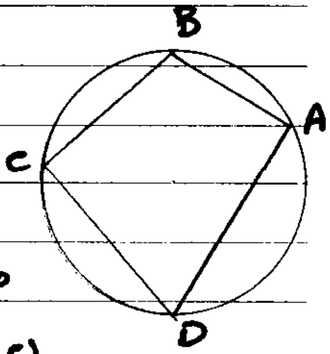
$\therefore n_1=n_2$

Geometry 3rd

Choose

(1) d	(2) a	(3) d	(4) a
(5) b	(6) a	(7) b	(8) d
(9) b	(10) c	(11) d	(12) a
(13) a	(14) d	(15) c	(16) d
(17) c	(18) a	(19) c	(20) d
(21) d	(22) d	(23) d	(24) b
(25) d	(26) c	(27) d	(28) b
(29) c	(30) d	(31) d	(32) c
(33) a	(34) b	(35) b	(36) a
(37) c	(38) b	(39) d	

(1)



$m(\angle B)+m(\angle D)$

$=\frac{1}{2}m(\widehat{ADC})$

$+\frac{1}{2}m(\widehat{ABC})$

$=\frac{1}{2}\times 360=180^\circ$

$\therefore m(\angle A)+m(\angle C)$

$=360-180=180^\circ$

(2) $\because \overleftrightarrow{BD}$ is a tangent

$\therefore m(\angle DBA)=m(\angle C) \rightarrow (1)$

$\because \overleftrightarrow{BD} \parallel \overleftrightarrow{XY}$

$\therefore m(\angle DBA)=m(\angle BXY)$ Alt $\rightarrow (2)$

From (1),(2) $\therefore m(\angle BXY)=m(\angle C)$

$\therefore$ AXYC is a cyclic quad.

إعداد / محمود أحمد محب

③ ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are tangents to the smaller circle

∴ AB = AC

∴ $2x - 3 = 15$ ∴ $\boxed{x = 9}$

∵ $\overrightarrow{AB}$, $\overrightarrow{AD}$ are tangents to the greater circle

∴ AB = AD

$y - 2 = 15$ $\boxed{y = 17}$

④ $m(\angle D) = m(\angle A) = 30^\circ$ subtended by $\widehat{BC}$

∴ $m(\widehat{BC}) = 2 \times 30 = 60^\circ$

∴ $m(\widehat{AD}) = \frac{180 - 60}{2} = 60^\circ$

∴ $m(\angle C) = \frac{1}{2} m(\widehat{AD}) = 30^\circ$

∵ $m(\angle C) = m(\angle A)$

But they are alternate.

∴ $\overline{AB} \parallel \overline{DC}$.

⑤ It's easy to answer:

① IF there are two opposite supplementary angles

② IF there is an exterior angle equal in measure to the measure of the opposite to its adjacent angle.

③ IF there are two angles equal in measure and drawn on one side and on one side of this side

⑥ ∵ x is the midpoint of $\overline{AB}$

∴ $\overline{Mx} \perp \overline{AB}$

∵ y is the midpoint of $\overline{AC}$

∴ $\overline{My} \perp \overline{AC}$

∴ $m(\angle M) = 360^\circ - (90 + 90 + 70) = \boxed{110^\circ}$

∵ AC = AB

∴ My = Mx , ∵ MH = Mx = r

∴ $\boxed{YH = xD}$

⑦ $m(\widehat{HC}) - m(\widehat{BD}) = 2m(\angle A)$

$120^\circ - m(\widehat{BD}) = 60$

∴ $m(\widehat{BD}) = \boxed{60^\circ}$

∴ $m(\widehat{HD}) = \frac{360 - (120 + 60)}{2} = 90^\circ$

∴ $m(\angle C) = \frac{1}{2} m(\widehat{HDB}) = 75^\circ$

$m(\angle H) = \frac{1}{2} m(\widehat{CBD}) = 75^\circ$

∴ HDBC is a cyclic quad.

∴ $m(\angle ADB) = m(\angle C) = 75^\circ$

∴ $m(\angle ABD) = m(\angle H) = 75^\circ$

∴ $\boxed{AD = AB}$

⑧ ∵ $\overrightarrow{DA}$ is a tangent

∴ $m(\angle DAB) = m(\angle C)$ → ①

∵ $\overrightarrow{DB}$ is a tangent

∴ $m(\angle DBA) = m(\angle C)$ → ②

∵ AB = AC

∴ $m(\angle ABC) = m(\angle C)$ → ③

From ①, ②, ③ we get

$m(\angle DAB) = m(\angle DBA) = m(\angle ABC) = m(\angle C)$

ΔΔ ADB , ABC in which $\begin{cases} m(\angle DBA) = m(\angle ABC) \\ m(\angle DAB) = m(\angle C) \end{cases}$

∴ $m(\angle BAC) = m(\angle BDA)$

∴ $\overline{AC}$ is a tangent to the circle ABD.

إعداد/ محمود احمد محي

(9) ∵ C is the midpoint of $\overline{AB}$ and MA = MB

∴ $\overrightarrow{MC} \perp \overline{AB}$, $\overrightarrow{MC}$ bisects (∠M)

∴ m(∠AMD) = 180 − (90 + 20) = 70°

∴ m(∠AMD) = m(∠BMD) = 70°

∴ m(∠BHD) = $\frac{1}{2}$ m(∠BMD) = **35°** subtended by ($\widehat{BD}$)

∴ m($\widehat{ADB}$) = m(∠AMB) = **140°**

(10) as no. (6)

(11) ∵ AB = AD

∴ m(∠ABD) = m(∠ADB) = 30°

∴ m(∠A) = 180 − (30 + 30) = 120°

∵ m(∠A) + m(∠C) = 180°

∴ ABCD is a cyclic quad.

(12) ∵ $\overline{BC}$ is a tangent

∴ m(∠CBD) = m(∠BAE) → (1) subtended by $\widehat{BE}$

∵ E is the midpoint of $\widehat{BF}$

∴ m($\widehat{BE}$) = m($\widehat{EF}$)

∴ m(∠FAE) = m(∠BAE) → (2)

From (1), (2), we get

m(∠CBD) = m(∠CAD)

∴ ABCD is a cyclic quad.

(13) ∵ $\overline{AD}$, $\overline{AF}$ are two tangents

∴ AD = AF = 5cm

similarly: BD = BE = 4cm

, CF = CE = 3cm

∴ P. of △ABC = 9 + 7 + 8 = 24cm

(14) As no. (2)

(15) ∵ $\overline{AC}$, $\overline{AB}$ are two tangents

∴ AB = AC

∴ m(∠ABC) = m(∠ACB) = $\frac{180-70}{2}$ = 55°

∵ BCDE is cyclic quade

∴ m(∠B) = 180 − 125 = 55°

∵ $\overline{CA}$ is a tangent

∴ m(∠ACB) = m(∠CEB) = 55° subtended by $\widehat{BC}$

∵ m(∠CEB) = m(∠CBE) = 55°

∴ **CB = CE**

∵ m(∠ACB) = m(∠CBE) = 55°

but the are alternate.

∴ $\overline{AC}$ // $\overline{BE}$.

(16) ∵ E is the midpoint of $\overline{XY}$

∴ $\overline{ME} \perp \overline{XY}$

∵ The two circles are intersecting at A, B

∴ $\overline{MN} \perp \overline{AB}$

∴ m(∠C) = 360 − (130 + 90 + 90) = **50°**

(17) ∵ $\overrightarrow{AB}$ is a tangent

∴ m(∠ABC) = m(∠D) = 70° subtended by $\widehat{BC}$

∵ $\overline{AB}$, $\overline{AC}$ are two tangents

∴ AB = AC

∴ m(∠ABC) = m(∠ACB) = 70°

∴ m(∠A) = 180 − (70 + 70) = **40°**

∵ CB = CD

∴ m(∠CBD) = m(∠CDB) = 70°

∴ m(∠ACB) = m(∠CBD)

∴ $\overline{AC}$ // $\overline{BD}$

(18) ∵ $\overline{BX} \parallel \overline{YC}$ ∴ $m(\widehat{XC}) = m(\widehat{BY})$

∴ $XC = BY$ ∴ $MA = MD$

(19) ∵ $m(\angle AEC) = m(\angle ADC) = 90°$

∴ AEDC is a cyclic quad.

∴ $m(\angle EAD) = m(\angle ECD)$

, ∵ $m(\angle BAX) = m(\angle BCX)$

subtended by $\widehat{BX}$

∴ $m(\angle BCE) = m(\angle BCX)$

∴ $\overrightarrow{CB}$ bisects $\angle ECX$

(20) ∵ ADEF is a cyclic quad.

∴ $m(\angle A) = 180 - 115 = 65°$

∴ $m(\angle DMF) = 2m(\angle A)$

$= 130°$

subtended by $\widehat{DF}$.

(21) half , subtended.

(22) As no.(13) $BE = 9$ cm

(23) ∵ $(\angle ABC)$ is an exterior

∴ $m(\angle ABC) = 25 + 40 = 65°$

∴ $m(\angle ADC) = m(\angle ABC) = 65°$

subtended by $\widehat{AC}$

(24) ∵ $\overline{AB}$ is a diameter

∴ $m(\angle ACB) = 90°$

∴ $m(\angle C) + m(\angle ADE) = 180°$

∴ ACED is cyclic quad.

∵ $(F\hat{E}C)$ is an exterior

∴ $m(\angle FEC) = m(\angle A)$ → ①

∵ $\overleftrightarrow{CF}$ is a tangent

∴ $m(\angle FCB) = m(\angle A)$ → ②

subtended by $\widehat{BC}$

from ①, ② we get

$m(\angle FCB) = m(\angle FEC)$

∴ $\triangle FCE$ is an isosceles.

(25) ∵ $AB = AD$

∴ $m(\angle ABD) = m(\angle ADB) = 70°$

∴ $m(\angle A) = 180 - (70 + 70) = 40°$

∴ $m(\angle C) = m(\angle A) = 40°$

subtended by $\widehat{DB}$.

$m(\angle ADC) = m(\angle ABC) = 20°$

subtended by $\widehat{AC}$

∴ $m(\angle BDC) = 70 + 20 = 90°$

(26) ∵ $m(\angle BAC) = m(\angle BDC) = 90°$

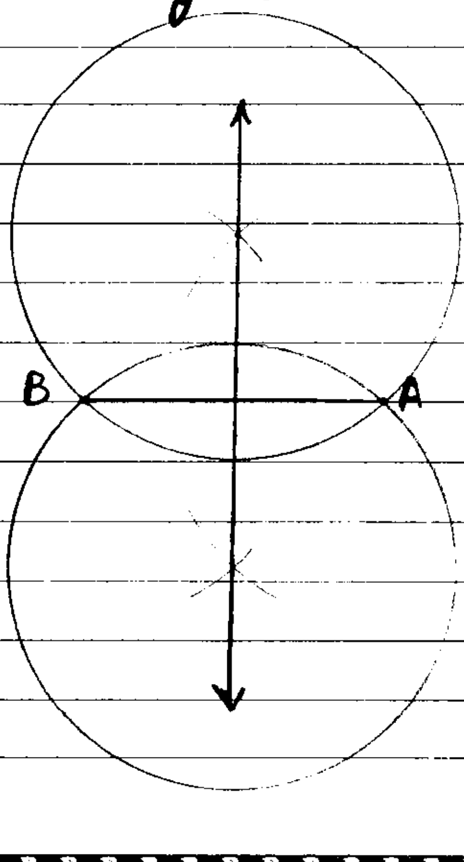
drawn on $\overline{BC}$

∴ ABCD is a cyclic quad.

∴ $m(\angle ABD) = \frac{1}{2} m(\angle AHD) = 24°$

subtended by $\widehat{AD}$.

(27)



إعداد/ محمود أحمد محي

(28) $D = C \div \pi = 44 \div \frac{22}{7} = 14$ cm

$\therefore AB = 14$ cm

$\because \overline{BC}$ is a tangent

$\therefore m(\angle B) = 90^\circ$, $\because m(\angle C) = 30^\circ$

$\therefore BC = 14\sqrt{3}$ cm.

(29) $\because m(\angle A) = 48^\circ$

$\therefore m(\widehat{BCD}) = 2 \times 48 = 96^\circ$

$\therefore m(\widehat{BD}$ the major$) = 360 - 96 = 264^\circ$

(30) $\because \overrightarrow{CA}$ is a tangent

$\therefore \overline{MA} \perp \overline{CA}$

$\because \overrightarrow{CB}$ is a tangent

$\therefore \overline{MB} \perp \overline{CB}$

$\because m(\angle A) + m(\angle B) = 180^\circ$

$\therefore$ AMBC is cyclic quad.

$\because (\angle DMB)$ is an exterior

$\therefore m(\angle DMB) = m(\angle ACB)$

(31) $\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore m(\widehat{DHC}) = m(\widehat{HDB})$

by subtracting $m(\widehat{HD})$

$\therefore m(\widehat{DB}) = m(\widehat{HC})$.

(32) $\because AB = AC$ $\therefore MX = NY \to ①$

$\because \overline{NY} \perp \overline{AD}$, $\overline{MX} \perp \overline{AD}$

$\therefore \overline{NY} \parallel \overline{MX} \to ②$

From ① ② we get

MXYN is a parallelogram

$\because m(\angle X) = 90^\circ$

$\therefore$ MXYN is a rectangle.

(34) Construction: Draw $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AE}$

Proof:

$\because m(\angle B) = m(\angle E)$

$\therefore AB = AE$

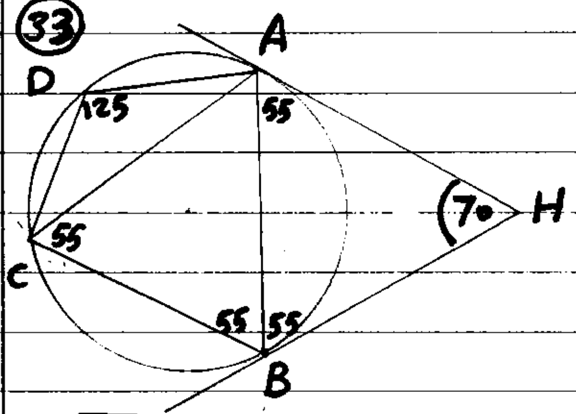
$\therefore MX = MY$

in the smaller circle

$\because MX = MY$

$\therefore CD = ZL$

(33)



$\because \overline{HA}, \overline{HB}$ are tangents

$\therefore HA = HB$.

$\therefore m(\angle HBA) = m(\angle HAB) = \frac{180 - 70}{2} = 55^\circ$

$\because \overrightarrow{HA}$ is a tangent

$\therefore m(\angle HAB) = m(\angle ACB) = 55^\circ$

subtended by $\widehat{AB}$

$\because$ ABCD is a cyclic quad.

$\therefore m(\angle ABC) = 180 - 125 = 55^\circ$

$\because m(\angle ABC) = m(\angle ACB)$

$\therefore$ **AB = AC**

$\therefore m(\angle BAC) = 180 - (55 + 55) = 70^\circ$

$\because m(\angle CAB) = m(\angle AHB)$

$\therefore \overleftrightarrow{AC}$ is a tangent to the circle passing through the points A, B and H.

إعداد/ محمود أحمد محي

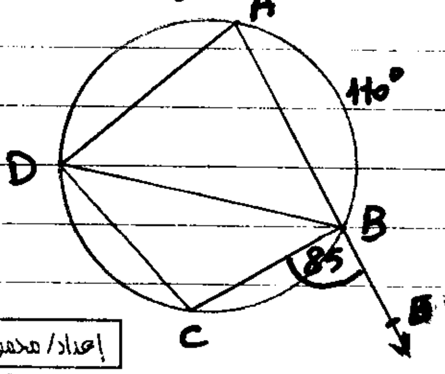
(35) ∵ $\overrightarrow{AB}$ is a tangent
∴ $\overline{MB} \perp \overline{AB}$
∴ $m(\angle ABM) = 90°$
∴ $m(\angle BMC) = 180 - (90 + 40) = 50°$
∴ $m(\angle BDC) = \frac{1}{2} m(\angle BMC) = 25°$
subtended by $\overset{\frown}{BC}$.

(36) ∵ $\overline{AB}$ diameter, $\overline{BY}$ tangent
∴ $m(\angle ABY) = 90°$
∵ X midpoint of $\overline{AC}$
∴ $\overline{MX} \perp \overline{AC}$
∴ $m(\angle AXY) = 90°$
∴ $m(\angle ABY) = m(\angle AXY)$
drawn in $\overline{AY}$
∴ XAYB is a cyclic quad.

(37) ∵ $\overline{XZ}, \overline{XY}$ are tangents
∴ XZ = XY
∴ $m(\angle XZY) = m(\angle XYZ)$
$= \frac{180 - 40}{2} = 70°$
∵ $\overrightarrow{XZ}$ is a tangent
∴ $m(\angle XZY) = m(\angle ZEY) = 70°$
subtended by $\overset{\frown}{ZY}$.
∵ ZYED is a cyclic quade
∴ $m(\angle ZYE) = 180 - 110 = 70°$
∴ $m(\angle ZYE) = m(\angle ZEY)$

(38) As no.(23)

(39)



∵ ABCD is a cyclic quad.
and $(\angle CBE)$ is exterior
∴ $m(\angle CBE) = m(\angle ADC) = 85°$
∵ $m(\overset{\frown}{AB}) = 110°$
∴ $m(\angle ADB) = \frac{1}{2} m(\overset{\frown}{AB}) = 55°$
∴ $m(\angle BDC) = 85 - 55 = 30°$

(40) ∵ $\overrightarrow{AD}$ is a tangent
∴ $(\angle DAB)$ supplements $(\angle B)$
∴ $m(\angle B) = 180 - 130 = 50°$

(41) As n.(6)

(42) $m(\overset{\frown}{AC}) - m(\overset{\frown}{DB}) = 2m(\angle E)$
$80° - m(\overset{\frown}{DB}) = 60$
∴ $m(\overset{\frown}{DB}) = 20°$
∵ $\overline{AB}$ is a diameter
∴ $m(\overset{\frown}{ACB}) = 180°$
∴ $m(\overset{\frown}{CD}) = 180 - (80 + 20) = 80°$

(43) ∵ MA = MB = r
∴ $m(\angle MAB) = m(\angle MBA) = 50°$
∴ $m(\angle M) = 180 - (50 + 50) = 80°$
∴ $m(\angle C) = \frac{1}{2} m(\angle M) = 40°$
subtended by $\overset{\frown}{AB}$.

(44) ∵ ABCD is a cyclic quad.
, $(\angle ABE)$ is an exterior
∴ $m(\angle ABE) = m(\angle D) = 100°$
∴ $m(\angle DCA) = 180 - (100 + 40) = 40°$
∵ $m(\angle DCA) = m(\angle DAC)$
∴ △ ADC is an isosceles.

(45) ∵ $\overline{AB}$, $\overline{AC}$ are tangents

∴ $AB = AC$

∴ $m(\angle ABC) = m(\angle ACB)$

$= \frac{180-50}{2} = 65^\circ$

∵ $\overline{AB}$ is a tangent

∴ $m(\angle ABC) = m(\angle BEC) = 65^\circ$

subtended by $\widehat{BC}$

∵ BCDE is a cyclic quad.

∴ $m(\angle EBC) = 180 - 115 = 65^\circ$

∴ $m(\angle ABC) = m(\angle CBE)$

∴ $\overrightarrow{BC}$ bisects $(\angle ABE)$

∵ $m(\angle CBE) = m(\angle CEB)$

∴ $CB = CE$

Best wishes

SECOND: GEOMETRY

Choose the correct answer:

(1) The line of centers of two intersecting circles is perpendicular to the common ………… and bisect it.

- (a) diameter
- (b) tangent
- (c) chord
- (d) arc

(2) The line of centers of two intersecting circles is the axis of symmetry of the common …………

- (a) diameter
- (b) tangent
- (c) chord
- (d) arc

(3) The measure of the inscribed angle drawn in a quarter of a circle = ………°

- (a) 135
- (b) 120
- (c) 90
- (d) 45

(4) The center of the inscribed circle of triangle is the intersection point of ……………

- (a) medians
- (c) altitudes
- (b) axes of its sides
- (d) bisectors of its angles

(5) The circumference of a circle is 8 π cm and a straight line is on a distance 3 cm from its center, then L is ………… the circle.

ⓐ outside ⓑ secant to ⓒ tangent to ⓓ otherwise

(6) If ABCD is a cyclic quadrilateral and $m(\angle A) = 3m(\angle C)$, then $m(\angle A)$ = ………°

ⓐ 180 ⓑ 135 ⓒ 90 ⓓ 45

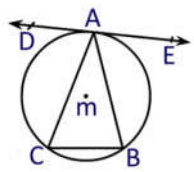
(7) M and N are two intersecting circles of radii lengths 6 cm and 4 cm, then MN ∈ …………

ⓐ]10,∞[ⓑ]2,10[ⓒ]0,2[ⓓ]4,6[

(8) In the opposite figure:

$\overleftrightarrow{ED}$ is a tangent, $m(\angle DAB) = 110^\circ$,

then $m(\angle C)$ …………



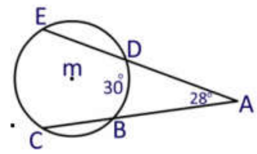
ⓐ 35 ⓑ 55 ⓒ 110 ⓓ 70

(9) A circle of radius length 5 cm, $\overline{AB}$ is a chord of length 8 cm, then the distance between the chord and the center = …… cm.

ⓐ 3 ⓑ 6 ⓒ 8 ⓓ 10

(10) From the opposite figure:

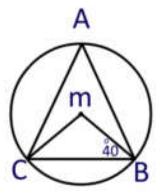
$m(arc\ EC)$ = ………°



ⓐ 56 ⓑ 30 ⓒ 86 ⓓ 28

(11) From the opposite figure:

$m(\angle A)$ = ………°



ⓐ 20 ⓑ 40 ⓒ 50 ⓓ 80

(12) The measure of the central angle in a circle ……… the measure of the inscribed angle subtended by the same arc.

ⓐ supplements ⓑ equal ⓒ half ⓓ double

(13) The length of an arc which represents a semicircle = ...

ⓐ π r   ⓑ 2 π r   ⓒ $\frac{1}{2}$ π r   ⓓ $\frac{1}{4}$ π r

(14) If AB = 6 cm, then the number of circles which passes through A and B of radius length 3 cm is ....

ⓐ 0   ⓑ 1   ⓒ 2   ⓓ infinite

(15) If AB = 5 cm, then the number of circles which passes through A and B of radius length 3 cm is ....

ⓐ 0   ⓑ 1   ⓒ 2   ⓓ infinite

(16) If AB = 8 cm, then the number of circles which passes through A and B of radius length 3 cm is ....

ⓐ 0   ⓑ 1   ⓒ 2   ⓓ infinite

(17) The number of common tangents of two distant circles is ....

ⓐ 1   ⓑ 2   ⓒ 3   ⓓ 4

(18) If the longest chord in a circle is 12 cm, then its circumference = ... cm

ⓐ 6 π ⓑ 12 π ⓒ 24 π ⓓ 144 π

(19) If the lengths of the radii of the two circles M and N are 6 cm, 8 cm and MN = 14 cm, then the two circles are ....

ⓐ intersecting
ⓑ distant
ⓒ touching externally
ⓓ one inside the other

(20) The inscribed angle in a semicircle is ....

ⓐ acute   ⓑ straight   ⓒ right   ⓓ obtuse

(21) A chord of length 8 cm drawn in a circle of diameter length 10 cm, then the distance between the chord and the center is .... cm.

ⓐ 3   ⓑ 4   ⓒ 5   ⓓ 6

(22) Number of tangents of two touching internally circles is ....

a 0 b 1 c 2 d 3

(23) If ABCD is a cyclic quadrilateral and $m(\angle A) = 2m(\angle C)$, then $m(\angle A)$ = ...°

a 30 b 60 c 90 d 120

(24) If the lengths of radii of two circles M and N are 6 cm, 3 cm and MN = 2 cm, then the two circles are ....

a intersecting
b distant
c one inside the other
d touching externally

(25) Circle of diameter length $2x$ cm, a straight line of distance $x+1$ cm from its center, then the straight line is .... circle.

a tangent to the
b axis of symmetry of the
c secant to the
d outside the

(26) Number of common tangents of a two concentric circles is ....

a 3 b 2 c 1 d 0

(27) The measure of the inscribed angle in a semicircle = ...°

a 360 b 180 c 120 d 90

(28) If the lengths of radii of two circles M and N are 9 cm, 4 cm and MN = 5 cm, then the two circles are ....

a intersecting
b distant
c touching internally
d touching externally

(29) The centers of circles which passes through the two points A and B lies on ....

a $\overline{AB}$
b midpoint of $\overline{AB}$
c axis of $\overline{AB}$
d perpendicular to $\overline{AB}$ at B

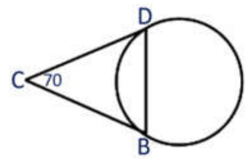
(30) The measure of the inscribed angle in a third of a circle = ...°

a 360 b 180 c 120 d 90

(31) The measure of the inscribed angle in a quarter of a circle is ………..°

ⓐ 45 ⓑ 90 ⓒ 135 ⓓ 145

(32) From the opposite figure:



$m(arc\,BD)$ = ……..°

ⓐ 55 ⓑ 90 ⓒ 180 ⓓ 110

(33) The length of an arc which represents a quarter of a circle of radius length r cm is ……… cm

ⓐ $4\pi r$ ⓑ $2\pi r$ ⓒ πr ⓓ $\frac{1}{2}\pi r$

(34) One of the following identifies a unique circle …………

ⓐ length of radius and a point
ⓑ two points
ⓒ one point
ⓓ center and a point

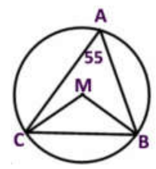
(35) Circle of diameter length 6 cm, a straight line of distance 6 cm from its center, then the straight line is ………………

ⓐ outside the circle
ⓑ tangent to the circle
ⓒ secant to the circle
ⓓ passes through the center

(36) If ABCD is a cyclic quadrilateral, $m(\angle A) = 90^o$, then the diameter of the circle is ……………

ⓐ $\overline{AB}$ ⓑ $\overline{AC}$ ⓒ $\overline{AD}$ ⓓ $\overline{BD}$

(37) From the opposite figure:



$m(\angle MCB)$ = …………°

ⓐ 110 ⓑ 35 ⓒ 45 ⓓ 55

(38) Number of axes of symmetry of two congruent circles and touching externally is ………………

ⓐ 3 ⓑ 2 ⓒ 1 ⓓ infinite

(39) Number of axes of symmetry of two touching externally circles is ………………

ⓐ 3 ⓑ 2 ⓒ 1 ⓓ infinite

(40) Number of common tangents of two touching externally circles is …………

ⓐ 3 ⓑ 2 ⓒ 1 ⓓ infinite

(41) Two touching circles of radii lengths 5 cm and 8 cm, then the distance between their centers $\in$ ……………

ⓐ]13,3[ⓑ]3,13[ⓒ R-[3,13] ⓓ {3,13}

(42) Two intersecting circles of radii lengths 5 cm and 3 cm, then the distance between their centers $\in$ ……………

ⓐ]8,∞[ⓑ]2,∞[ⓒ]0,2] ⓓ]2,8[

(43) We can't draw a circle passing through the vertices of ………

ⓐ triangle ⓑ rectangle ⓒ rhombus ⓓ square

(44) The minor arc in the circle is opposite to ………… inscribed angle.

ⓐ an acute ⓑ an obtuse ⓒ a right ⓓ a reflex

(45) The radius length of the smallest angle passing through the endpoints of a line segment ………… half of its length.

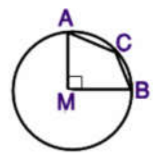
ⓐ less than ⓑ more than ⓒ equals ⓓ double

(46) The two tangents to a circle drawn from the endpoints of its diameter are …………………

ⓐ parallel ⓑ equal ⓒ coincident ⓓ intersecting

(47) From the opposite figure:

$m(\angle ACB)$ = ………°

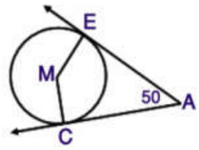


ⓐ 45 ⓑ 110 ⓒ 135 ⓓ 270

(48) The measure of an arc which represents a third of a circle is ………°

ⓐ 60 ⓑ 90 ⓒ 120 ⓓ 240

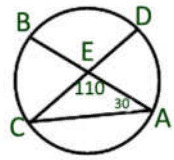
(49) From the opposite figure:



$m(arc\ EC) =$ …………°

ⓐ 100 ⓑ 120 ⓒ 130 ⓓ 50

(50) From the opposite figure:



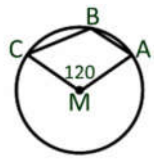
$m(arc\ DA) =$ …………°

ⓐ 40 ⓑ 55 ⓒ 80 ⓓ 110

(51) If AB = 6 cm, then the area of the smallest circle passing through A and B is ……………… cm^2.

ⓐ 3 π ⓑ 6 π ⓒ 8 π ⓓ 9 π

(52) From the opposite figure:



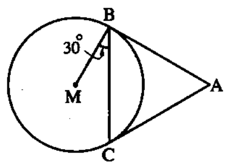
$m(\angle ABC) =$ …………°

ⓐ 60 ⓑ 120 ⓒ 240 ⓓ 360

(53) The center of the circumcircle of a triangle is the intersection point of ……………………

ⓐ its medians
ⓑ axes of its sides
ⓒ its altitudes
ⓓ bisectors of its angles

(54) From the opposite figure:



$m(\angle BAC) =$ …………°

ⓐ 90 ⓑ 60 ⓒ 30 ⓓ 15

Essay Problems:

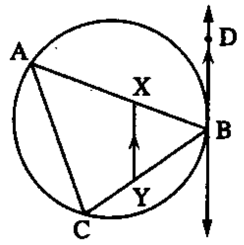
(1) **In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, $X \in \overline{AB}$, $Y \in \overline{BC}$ where $\overline{XY} // \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



(2) **In the opposite figure :**

Two circles are touching internally at B

, $\overrightarrow{AB}$ is a common tangent

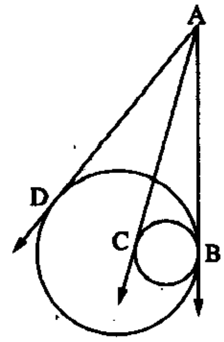
, $\overrightarrow{AC}$ is a tangent to the smaller circle at C

, $\overrightarrow{AD}$ is a tangent to the greater circle at D

, AC = 15 cm. , AB = $(2x - 3)$ cm.

and AD = $(y - 2)$ cm.

Find : The value of each of x and y



(3) **In the opposite figure :**

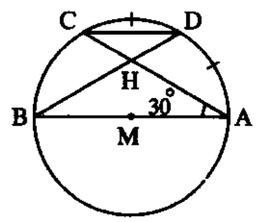
$\overline{AB}$ is a diameter in the circle M

, C ∈ the circle M , m (∠ CAB) = 30°

, D is midpoint of $\overset{\frown}{AC}$, $\overline{DB} \cap \overline{AC} = \{H\}$

(1) **Find :** m (∠ BDC) and m $(\overset{\frown}{AD})$

(2) **Prove that :** $\overline{AB} // \overline{DC}$



(4) **In the opposite figure :**

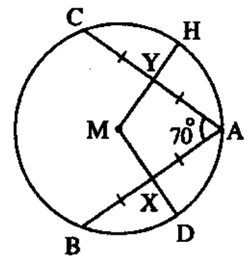
$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in circle M

, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$

, m (∠ CAB) = 70°

(1) **Calculate :** m (∠ DMH)

(2) **Prove that :** XD = YH



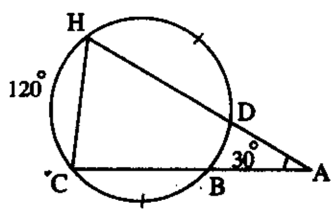
(5) **In the opposite figure :**

m (∠ A) = 30° , m $(\overset{\frown}{HC})$ = 120°

, m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{DH})$

(1) **Find :** m $(\overset{\frown}{BD}$ the minor)

(2) **Prove that :** AB = AD



(6) State two cases of a cyclic quadrilateral.

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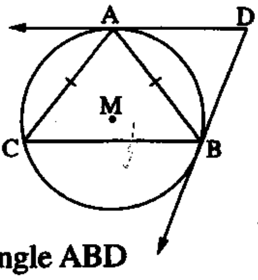
(7) **In the opposite figure :**



$\overrightarrow{DA}$ and $\overrightarrow{DB}$ are two tangents of the circle M

and AB = AC

Prove that :

$\overline{AC}$ is a tangent to the circle passing through the vertices of the triangle ABD

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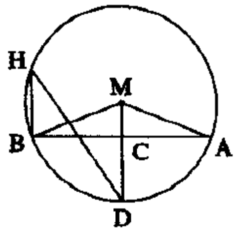
(8) **In the opposite figure :**



C is the midpoint of $\overline{AB}$, $\overrightarrow{MC} \cap$ the circle M = {D}

, m (∠ MAB) = 20°

Find : m (∠ BHD) and m ($\overset{\frown}{ADB}$)

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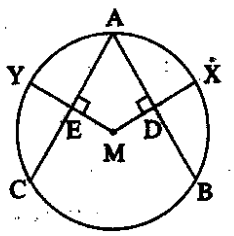
(9) **In the opposite figure :**



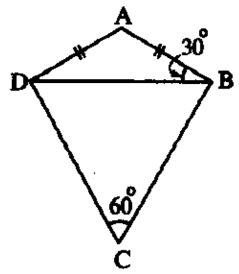
AB = AC , $\overline{MD} \perp \overline{AB}$,

$\overline{ME} \perp \overline{AC}$

Prove that : XD = YE

---

(10) **In the opposite figure :**



ABCD is a quadrilateral in which AB = AD ,

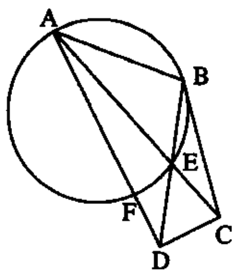
m (∠ ABD) = 30° ,

m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

---

(11) **In the opposite figure :**



$\overline{BC}$ is a tangent at B ,

E is the midpoint of $\overset{\frown}{BF}$

Prove that :

ABCD is a cyclic quadrilateral.

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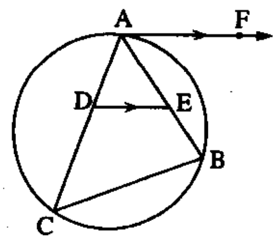
(12) **In the opposite figure :**



$\overrightarrow{AF}$ is a tangent to the circle at A , $\overrightarrow{AF}$ // $\overline{DE}$

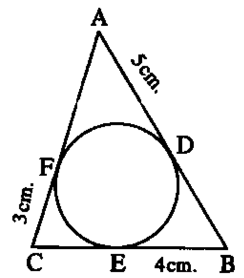
Prove that :

DEBC is a cyclic quadrilateral.

(13) **In the opposite figure :**

A circle is drawn touches the sides of a triangle ABC , $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F , AD = 5 cm , BE = 4 cm. , CF = 3 cm.

Find the perimeter of Δ ABC

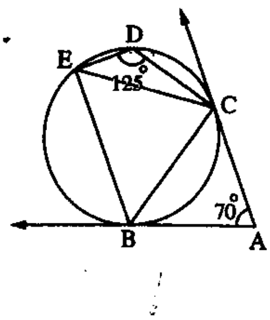


(14) **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the Circle at B , C , m (∠ A) = 70° , m (∠ CDE) = 125°

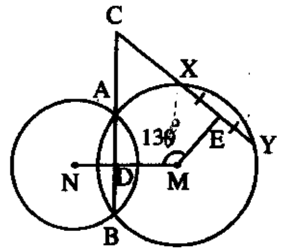
Prove that :

(1) CB = CE (2) $\overrightarrow{AC}$ // $\overline{BE}$



(15) **In the opposite figure :**

If E is the midpoint of $\overline{XY}$, m (∠ EMN) = 130° , **then find :** m (∠ C)

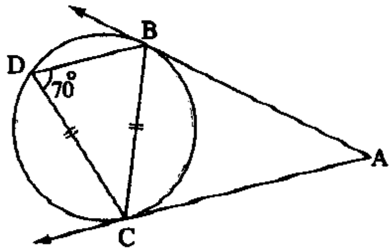


(16) **In the opposite figure :**

If $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C , m (∠ D) = 70° , CB = CD

(1) **Find :** m (∠ A)

(2) **Prove that :** $\overline{BD}$ // $\overline{AC}$

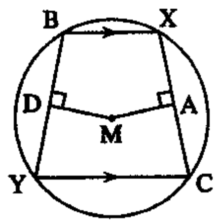


(17) **Complete :** The measure of the inscribed angle equals .... the measure of the central angle .... by the same arc.

(18) **In the opposite figure :**

$\overline{XB}$ // $\overline{CY}$, $\overline{MA} \perp \overline{XC}$, $\overline{MD} \perp \overline{BY}$

Prove that : MA = MD



(19)

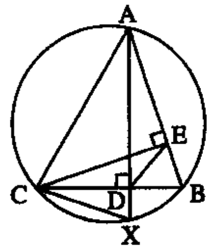
In the opposite figure :

$\overline{CE} \perp \overline{AB}$, $\overrightarrow{AD} \perp \overline{BC}$ and intersects the circle at X

Prove that :

(1) AEDC is a cyclic quadrilateral.

(2) $\overrightarrow{CB}$ bisects ∠ ECX

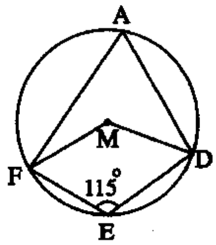


(20)

In the opposite figure :

If m (∠ DEF) = 115°

, then find : m (∠ DMF)



(21)

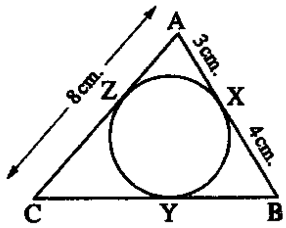
In the opposite figure :

Inscribed circle of the triangle ABC touches

its sides at X , Y and Z

If AX = 3 cm. , XB = 4 cm. , AC = 8 cm.

Find : The length of $\overline{BC}$



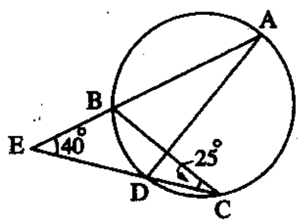
(22)

In the opposite figure :

$\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ C) = 25°

, m (∠ E) = 40°

Find : m (∠ ADC)



(23)

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

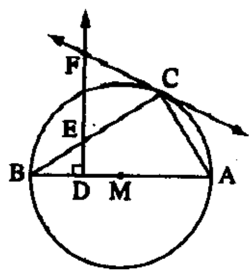
, $\overleftrightarrow{CF}$ is a tangent to the circle at C

, $\overrightarrow{DF} \perp \overline{AB}$ and intersects $\overline{BC}$ at E

Prove that :

(1) ADEC is a cyclic quadrilateral.

(2) Δ FCE is an isosceles triangle.



(24)

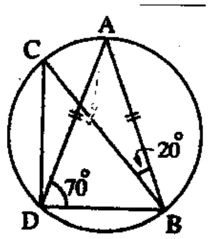
In the opposite figure :

AB = AD

, m (∠ ABC) = 20°

, m (∠ ADB) = 70°

Find : m (∠ C) , m (∠ BDC)



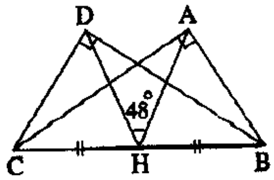
(25) **In the opposite figure :**

m (∠ BAC) = m (∠ BDC) = 90°

, H is the midpoint of $\overline{BC}$ and m (∠ AHD) = 48°

(1) **Prove that :** ABCD is a cyclic quadrilateral.

(2) **Find :** m (∠ ABD)



(26) Using your geometric tools , draw $\overline{AB}$ with a length of 4 cm. , then draw a circle passing through the two points A and B whose radius length is 3 cm.

What are the possible solutions ? (Don't remove the arcs)

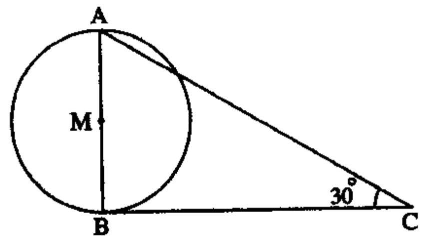
(27) **In the opposite figure :**

A circle M of circumference 44 cm.

, $\overline{AB}$ is a diameter , $\overline{BC}$ is a tangent at B

and m (∠ ACB) = 30°

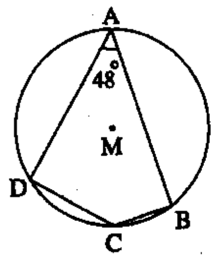
Find : The length of $\overline{BC}$ $\left(\pi = \frac{22}{7}\right)$



(28) **In the opposite figure :**

If M is a circle , m (∠ A) = 48°

Find : m ($\overset{\frown}{BD}$ the major)



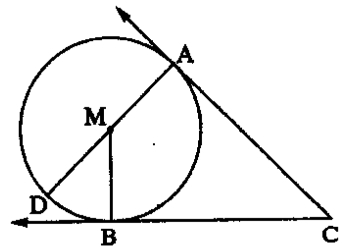
(29) **In the opposite figure :**

$\overline{AD}$ is a diameter in a circle M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle M ,

touch it at A and B respectively.

Prove that : m (∠ DMB) = m (∠ ACB)



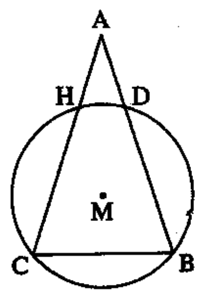
(30) **In the opposite figure :**

ABC is a triangle in which AB = AC

, $\overline{BC}$ is a chord in the circle M

, if $\overline{AB}$ and $\overline{AC}$ cut the circle at D and H respectively.

Prove that : m ($\overset{\frown}{DB}$) = m ($\overset{\frown}{HC}$)



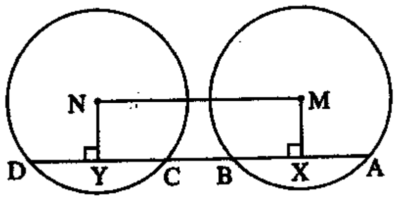
(31)

In the opposite figure :

M and N are two congruent circles

, AB = CD

Prove that : The figure MXYN is a rectangle.



(32)

ABCD is a quadrilateral inscribed in a circle , H is a point outside the circle and $\overrightarrow{HA}$ and $\overrightarrow{HB}$ are two tangents to the circle at A and B , if m (∠ AHB) = 70° and m (∠ ADC) = 125° , **prove that :**

(1) AB = AC

(2) $\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and H

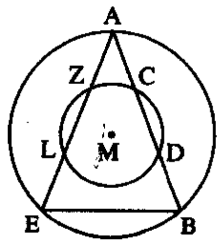
(33)

In the opposite figure :

Two concentric circles at M

, m (∠ ABE) = m (∠ AEB)

Prove that : CD = ZL



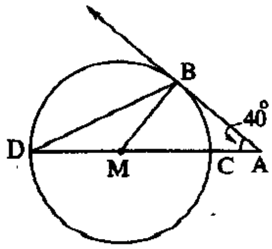
(34)

In the opposite figure :

$\overrightarrow{AB}$ is a tangent to the circle M

, m (∠ A) = 40°

Find with proof : m (∠ BDC)



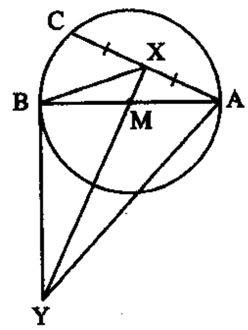
(35)

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{AC}$ and $\overrightarrow{XM}$ intersecting the tangnet of the circle at B in Y

Prove that : The figure AXBY is a cyclic quadrilateral.



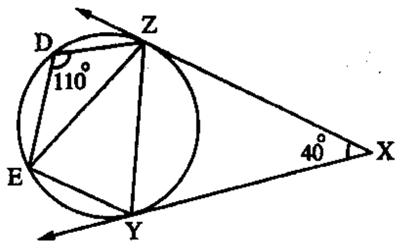
(36)

In the opposite figure :

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle at the two points Y and Z , m (∠ X) = 40°

, m (∠ D) = 110°

Prove that : m (∠ ZYE) = m (∠ ZEY)



(37)

ABCD is a quadrilateral drawn in a circle , $E \in \overrightarrow{AB}$, $E \notin \overline{AB}$

, m $(\overset{\frown}{AB})$ = 110° , m (∠ CBE) = 85°

Find with proof : m (∠ BDC)

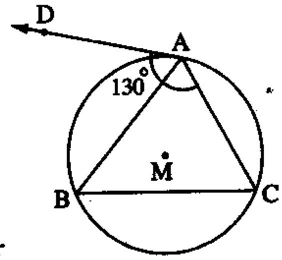
(38)

In the opposite figure :

$\overrightarrow{AD}$ is the tangent to the circle M at A

, m (∠ DAC) = 130°

Find with proof : m (∠ B)



(39)

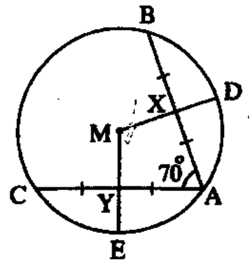
In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length at the circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ A) = 70°

(1) **Find :** m (∠ DME) (2) **Prove that :** XD = YE



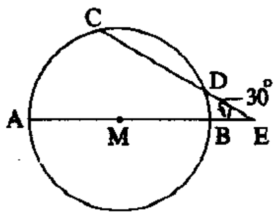
(40)

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ E) = 30° , m $(\overset{\frown}{AC})$ = 80°

Find : m $(\overset{\frown}{CD})$

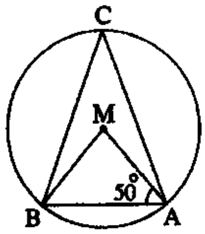


(41)

In the opposite figure :

M is a circle , m (∠ MAB) = 50°

Find : m (∠ C)



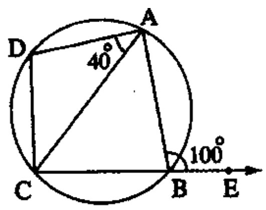
(42)

In the opposite figure :

m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : Δ DAC is an isosceles triangle.



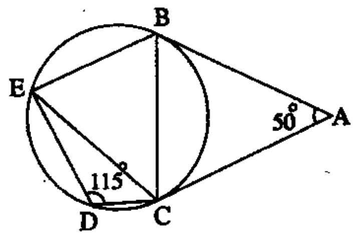
(43)

In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m (∠ A) = 50° , m (∠ D) = 115°

Prove that : (1) $\overrightarrow{BC}$ bisects ∠ ABE (2) CB = CE



(44)

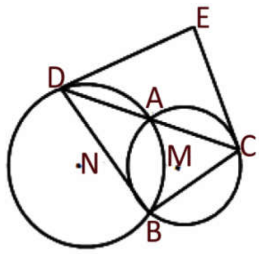
In the opposite figure:

M, N are two intersecting circles in A , B

$\overrightarrow{EC}$ is tangent to the circle M at C,

$\overrightarrow{DC}$ is tangent to the circle N at D,

Prove that: ECDB is cyclic quadrilateral

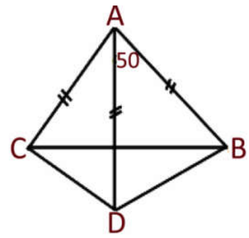


(45)

In the opposite figure:

A B = AC = AD, m (∠ BAD) = 50°

Find m (∠ BCD)



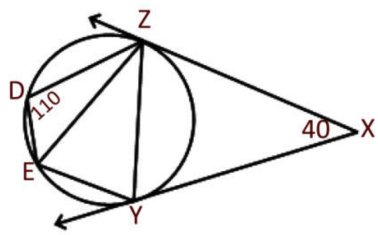
(46)

In the opposite figure:

$\overline{XY}$, $\overline{XZ}$ are two tangents to the circle

M (∠ YXZ) = 40° , m (∠ Z D E) = 110°

Prove that: Z E = Z Y

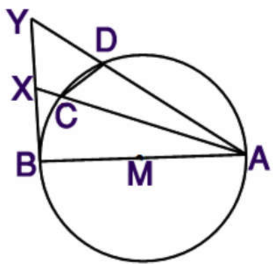


(47)

In the opposite figure:

$\overline{AB}$ is a diameter in circle M, $\overline{YB}$ is tangent.

Prove that: DCXY is cyclic quadrilateral



1) Choose the correct answer

1) If M circle with radius length = 4 cm and A is a point in its plane, MA = 3 cm, then A is .... circle M.

(inside – on – outside)

2) If M circle with radius length = 4 cm and A is a point in its plane, MA = 4 cm, then A is .... circle M.

(inside – on – outside)

3) If M circle with radius length = 4 cm and A is a point in its plane, MA = 5 cm, then A is .... circle M.

(inside – on – outside)

4) A tangent to a circle is ....the radius at its point of tangency.

(perpendicular to – parallel to – bisects)

5) If a straight line is perpendicular to a diameter of a circle at one of its endpoints, then it is a .... to the circle.

(axis of symmetry – tangent – chord)

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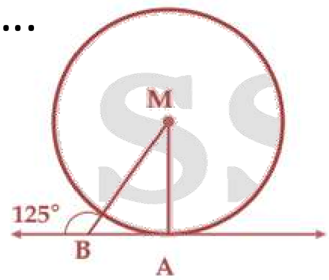
6) In the opposite figure: $m\ (\angle AMB\ )$ = ...

(25° – 35° – 45°)



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7) If the surface of the circle M ∩ If the surface of the circle N = Ø , then the two circles are ....

(Distant - touching externally - intersecting)

8) If M and N are two centers of two circles with radii r_1 , r_2 , where MN > $r_1 + r_2$, then the two circles are ....

(Distant - touching externally - intersecting)

9) If the surface of the circle M ∩ If the surface of the circle N = $\{A\}$, then the two circles are ……………………

(touching externally - touching internally - intersecting)

10) If the surface of the circle M ∩ If the surface of the circle N = the surface of the circle N , then the two circles are ……………

(Distant - touching externally - one inside the other)

11) M and N are two circles touching externally , their radii 9cm , 4cm , then MN = …………..cm (5cm - 7 cm - 13 cm)

12) M and N are two circles touching internally , their radii 9cm , 4cm , then MN = …………..cm (5cm - 7 cm - 12 cm)

13) M and N are two circles, their radii 7cm , 5cm , then MN = 12cm , then the two circles are ……………………

(Distant - touching externally - touching internally)

14) M and N are two circles, their radii 7cm , 5cm , then MN = 2cm , then the two circles are ……………………

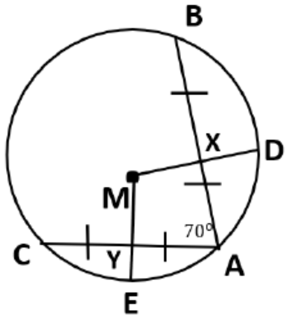
(Distant - touching externally - touching internally)

15) M and N are two circles, their radii 7cm , 5cm , then MN = 15cm , then the two circles are ……………………

(Distant - touching externally - touching internally)

16) M and N are two intersecting circles their radii 4cm and 6cm then MN ∈ ………….. **( ]2, 5[,]2, 10[,]4, 9[**

2) In the opposite figure:



AB and AC are two equal chords in circle M , X and Y are the midpoint of AB and AC m(∠A) = 70°

a) Find m(∠DME)

b) Prove that XD = YE

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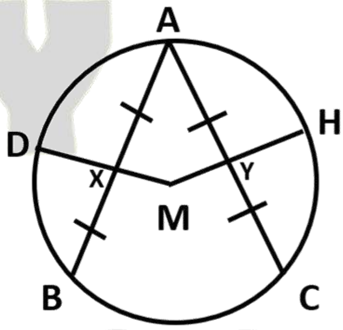
3) In the opposite figure:



AB = AC , X is the mid-point of $\overline{AB}$, Y is the mid-point of $\overline{AC}$ **prove that: DX = HY**

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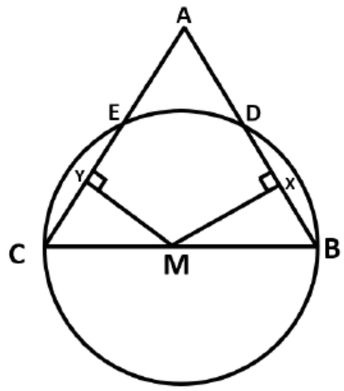
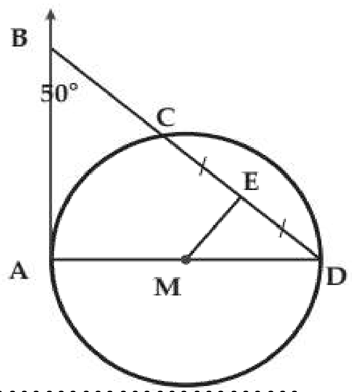
4) In the opposite figure:



ABC is a triangle in which AB = AC. circle M was drawn with diameter $\overline{BC}$ intersecting $\overline{AB}$ at D and $\overline{AC}$ at E , $\overline{MX} \perp \overline{BD}$, $\overline{MY} \perp \overline{CE}$ **prove that : BD = CE**

5) In the opposite figure:



AB is a tangent to the circle M, E is the midpoint of the chord CD , m (∠ ABC) = 50°

Find : $\boldsymbol{m\,(\angle AME\,)}$

6) In the opposite figure:

AB is a tangent to the circle M at A and
AM = 8 cm , m (∠ ABM) = 30°

***Find* the length of each : $\overline{AB}$ and $\overline{AC}$**

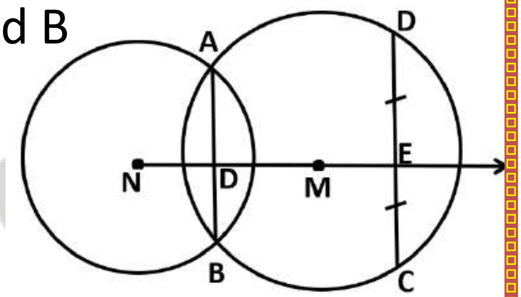
7) In the opposite figure:

The two circles M and N intersects at A and B

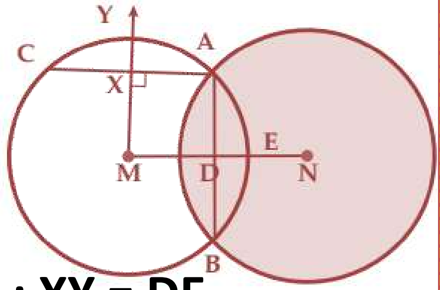
CD is a chord in the circle M cuts MN at E

, If E is the midpoint of CD

Prove that $\overline{AB}$ // $\overline{CD}$



8) In the opposite figure:



The two circles M and N intersect at A and B.

is drawn MX ⊥ AC MN is drawn , AC = AB

1) **Prove that : MD = MX** *2)* **Prove that : XY = DE**

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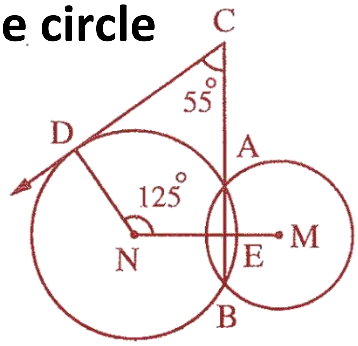
9) In the opposite figure:

M and N are two intersecting circles At A and B , m(∠C)=55° ,

m(∠N)=125° **Prove that :** $\overrightarrow{CD}$ **is a tangent to the circle**



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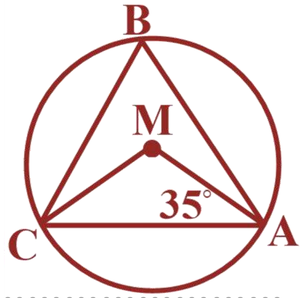
10) In the opposite figure:

M is a circle , m (∠ MAC) = 35°

Find m (∠ ABC)



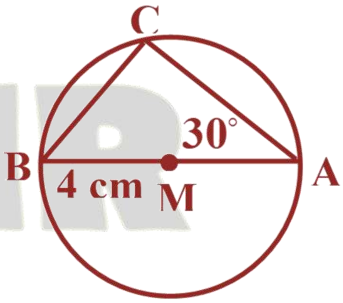
11) In the opposite figure:

$\overline{AB}$ is a diameter in the circle M

with radius length 4 cm , m (∠ A) = 30°

1) **Find m (∠ ABC)**

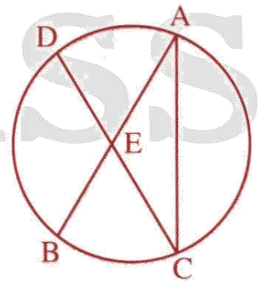
2) **Find the length of BC**



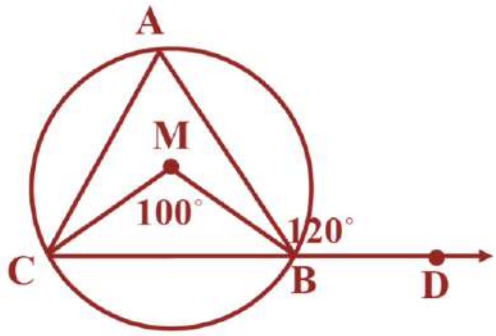
12) In the opposite figure:

AB and CD are two equal chords

Prove that ⊿ AEC is isosceles



13) In the opposite figure:



Δ ABC drawn in the circle M

D ∈ $\overrightarrow{CB}$ such that m (∠ ABD)=120°

if m (∠ BMC)=100°

Find with proof m (∠ ACB)

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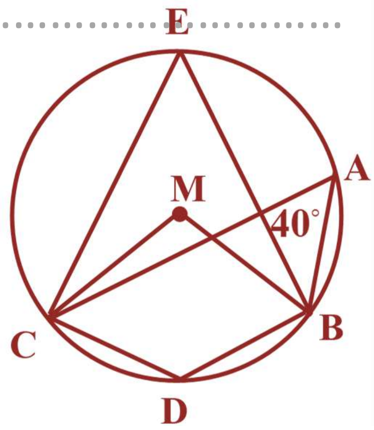
14) In the opposite figure:



The chords $\overline{AC}$ and $\overline{BE}$ intersects

At X , M is the centre of the circle,

if m(∠BAC)=40°

Find:

1) **m(∠BEC)** *2)* **m(∠BMC)**

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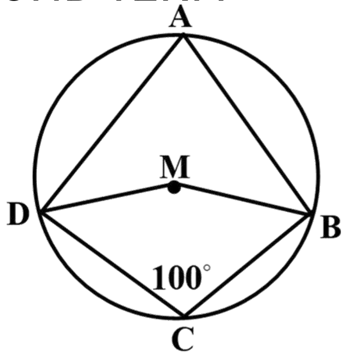
15) In the opposite figure:



M is a circle ABCD is a cyclic quadrilateral ,

m (∠ C) = 100°

Find : 1) $m(\angle A)$ 2) $m(\overset{\frown}{BCD})$

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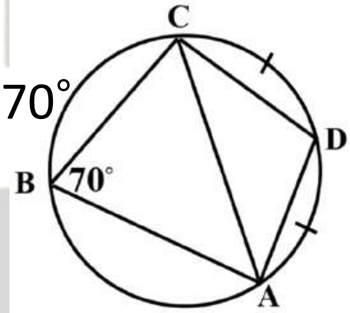
16) In the opposite figure:



ABCD is a cyclic quadrilateral in which m(∠ABC)=70°

The length of $\overset{\frown}{AD}$ = The length of $\overset{\frown}{DC}$

Find : m(∠ACD)

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17) Mention conditions of cyclic quadrilateral

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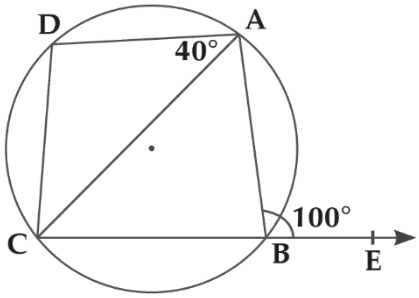
18) In the opposite figure:



m (∠ ABE) = 100° , m (∠CAD) = 40°

Prove that : m ($\widehat{CD}$) = m ($\widehat{AD}$) .

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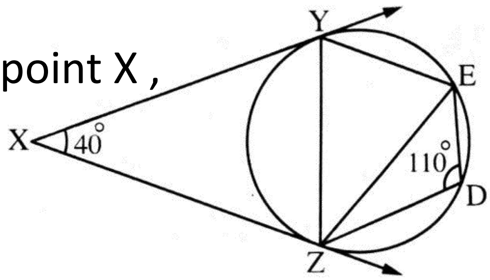
19) In the opposite figure:



XY and XZ are two tangents to the circle from point X ,

M(∠D) = 110° , M(∠X) = 40°

Prove that : m ($\widehat{ZE}$) = m ($\widehat{ZY}$) .

Best Wishes

MR.AMR ALFEKY

Qowesna, Monofia

3rd prep **Final revision** SECOND TERM

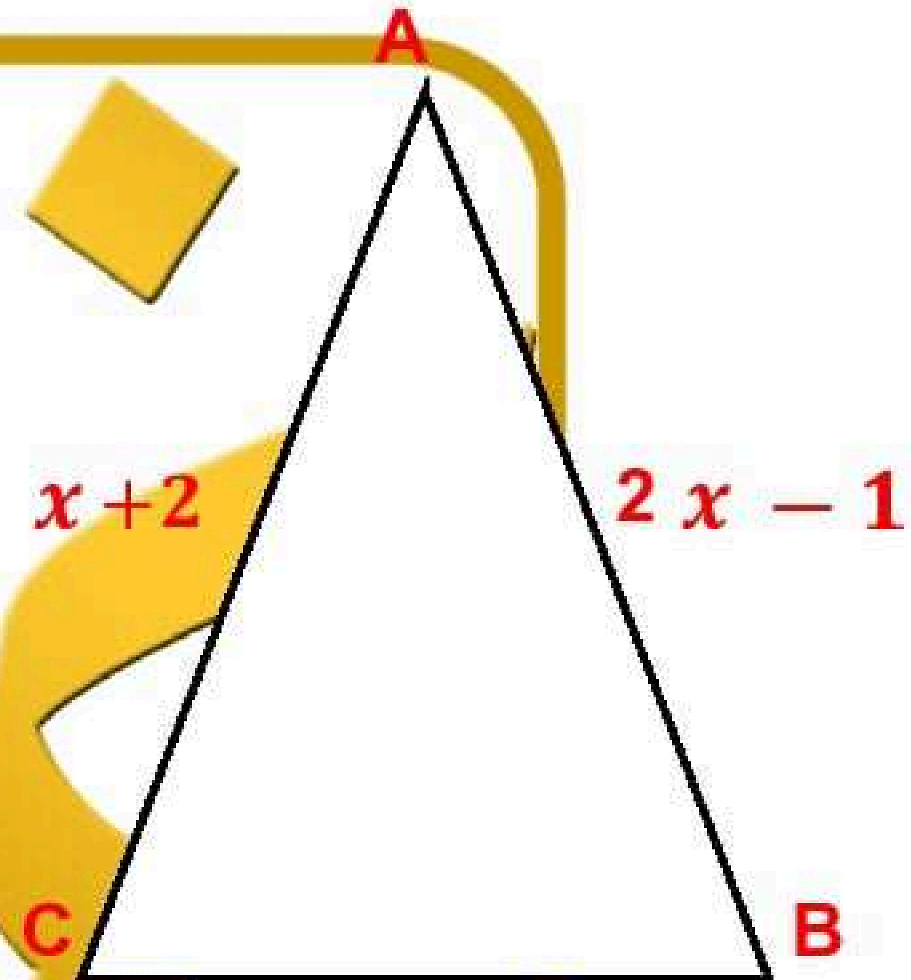
Geometry

Choose the correct answer:

(1) In the opposite figure:

AB = AC , AB = 2 $x-1$ and AC = $x+2$,

Then x = ………….



(a) 3 (b) 5 (c) 11 (d) 14

(1) M and N are two intersecting circles the lengths of their radii are 3 cm and 5 cm, then MN ∈ …………

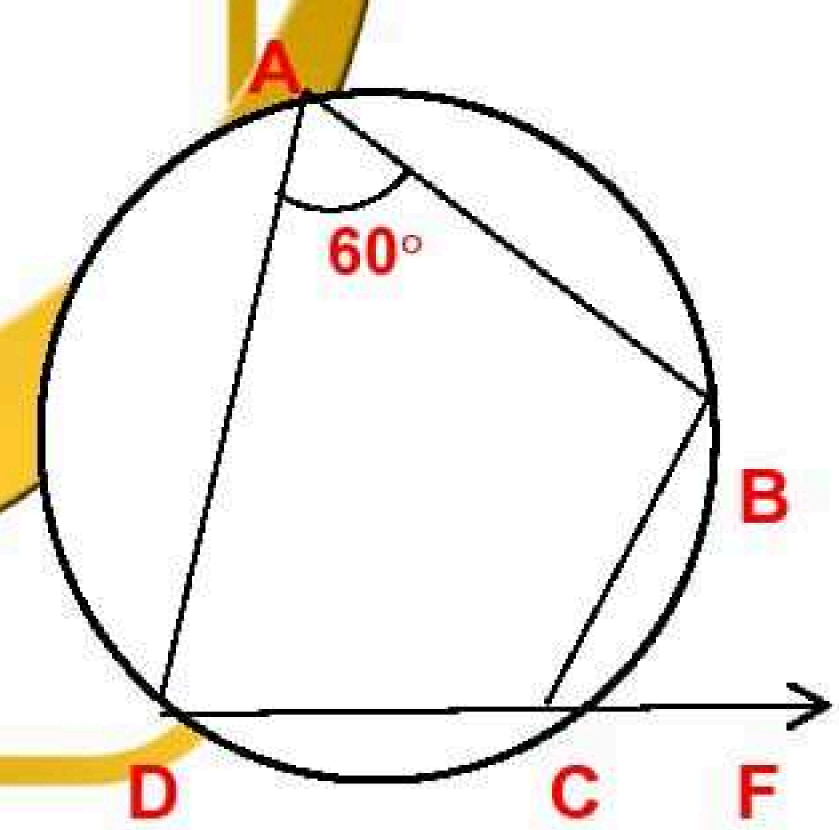
(a) [2 , 8] (b) [2 , 8 [(c)] 2 , 8] (d) [2 , 8]

(2) Number of the axes of symmetry of the semicircle is ……….

(a) zero (b) 1 (c) 2 (d) infinite

(3) In the opposite figure:

if m (∠ BAD) =60° , then m (∠ BCF) = ……..°



(a) 30 (b) 60 (c) 80 (d) 120

(5) The number of circles that pass through three collinear points equals ……….

(a) zero (b) one (c) three (d) infinite number

(6) The inscribed angle which opposite to the minor arc in a circle is …………

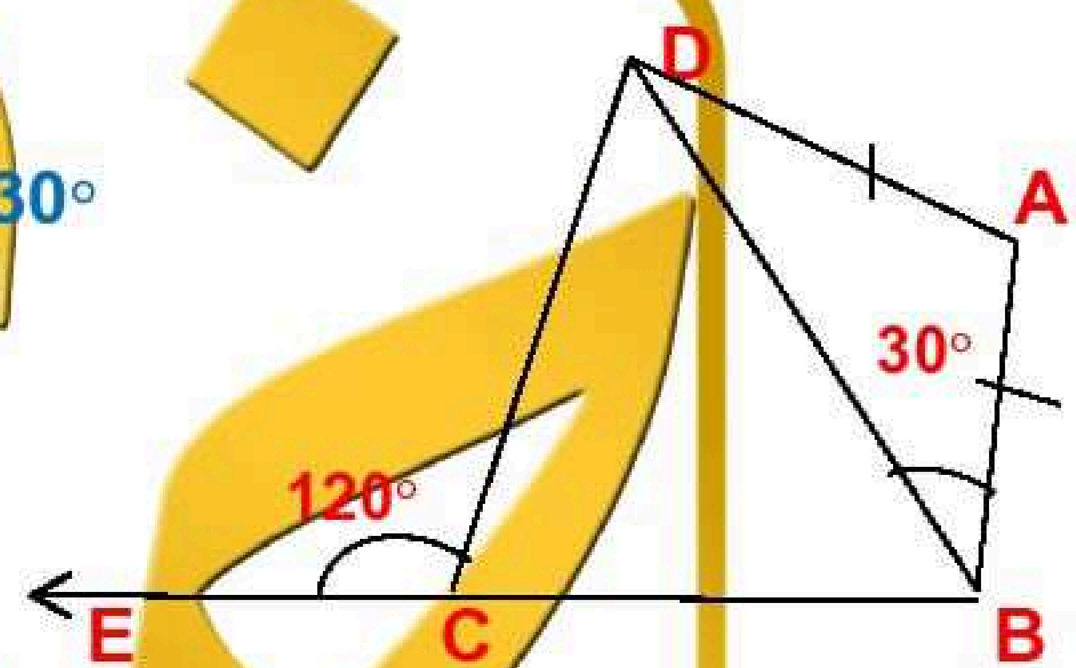
(a) reflex (b) right (c) obtuse (d) acute

(7) In the opposite figure :

ABCD is quadrilateral, m (∠ ABD) = 30°

m (∠ DCE) = 120°

then ABCD is ……….



(a) a rectangle (b) a rhombus

(c) a cyclic quadrilateral (d) a parallelogram

(8) The ratio between the measure of the inscribed angle and the measure of the central angle subtended by the same arc equals …..

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

(9) The area of a rhombus which the lengths of its diagonals are 6 cm, 8 cm equals ……….

(a) 2 cm^2 (b) 14 cm^2 (c) 24 cm^2 (d) 48 cm^2

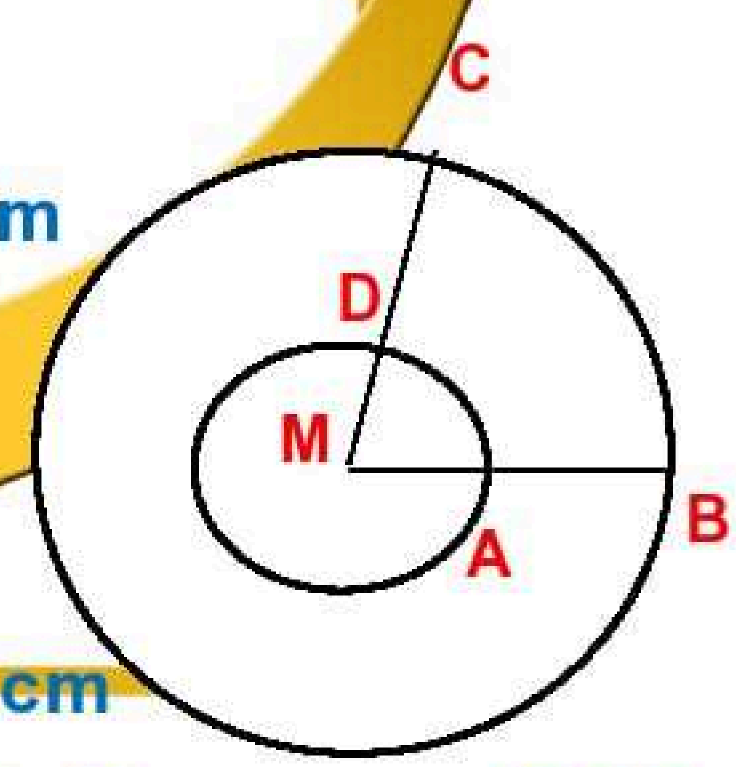
(10) In the opposite figure:

Two concentric circle M , m $\overset{\frown}{BC}$ = 80°

If the radius length of the smaller circle is 7 cm

and the radius length of the large circle is

14 cm, ($\pi = \frac{22}{7}$) then ;



First: the perimeter of the smaller circle = …. cm

(a) 44 (b) 22 (c) 154 (d) 88

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Second: m $(\overset{\frown}{AD})$ = ……..°

(a) 80 (b) 40 (c) 20 (d) 160

(11) The area of a square diagonal length is 6 cm equals …….. cm^2

(a) 36 (b) 18 (c) 24 (d) 9

(12) The length of the arc which represents $\frac{1}{4}$ of the perimeter of the circle = ……………

(a) $2\pi r$ (b) πr (c) $\frac{1}{2}\pi r$ (d) $4\pi r$

(13) AB is a line segment, then the number of the circles passing through the two points A , B is ……….

(a) 1 (b) 2 (c) 3 (d) infinite number

(14) If the straight line L ⋂ the circle M = ∅ , then L is ……… of the circle

(a) a secant (b) outside

(c) a tangent (d) an axis of symmetry

(15) If A and B are two points in the plane, if AB = 4 cm, then the smallest radius length of circle passing through by A and B is … cm

(a) 2 (b) 3 (c) 4 (d) 5

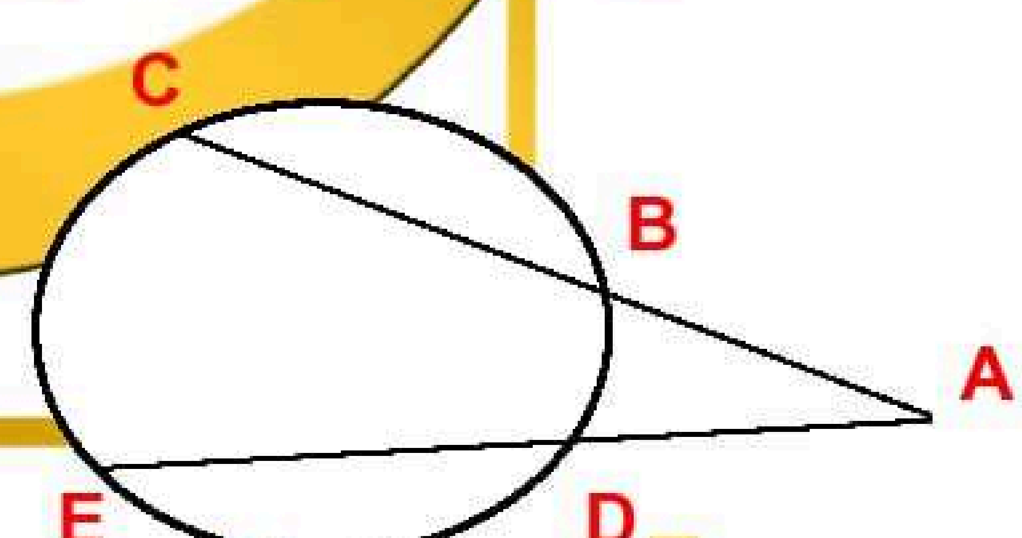
(16) If a straight line L is outside a circle of radius length 3 cm, and its center is the origin point M (0 , 0), If L at distance x from its center , then x ∈ ………….

(a) [3 , ∞ [(b)] 3 , ∞ [(c) [6 , ∞ [(d)] −∞ , −6 [

(17) In the opposite figure:

m $(\overset{\frown}{CE})$ = 100° , m $(\overset{\frown}{BD})$ = 30° ,

then m (∠ A) = ……..°



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(a) 70 (b) 65 (c) 50 (d) 35

(18) The number of axes of symmetry of two congruent circles and touching externally equals …………

(a) 4 (b) 2 (c) 1 (d) infinite number

(19) If the diameter length of a circle is 6 cm and the straight line L is distant from center by 6 cm, then L is …………..

(a) distant from the circle (b) intersects the circle

(c) touches the circle (d) passes through the center of circle

(20) If DHWQ is a cyclic quadrilateral with a right-angle at vertices Q , then ………… is a diameter in its circle

(a) $\overline{DQ}$ (b) $\overline{HW}$ (c) $\overline{WD}$ (d) $\overline{DH}$

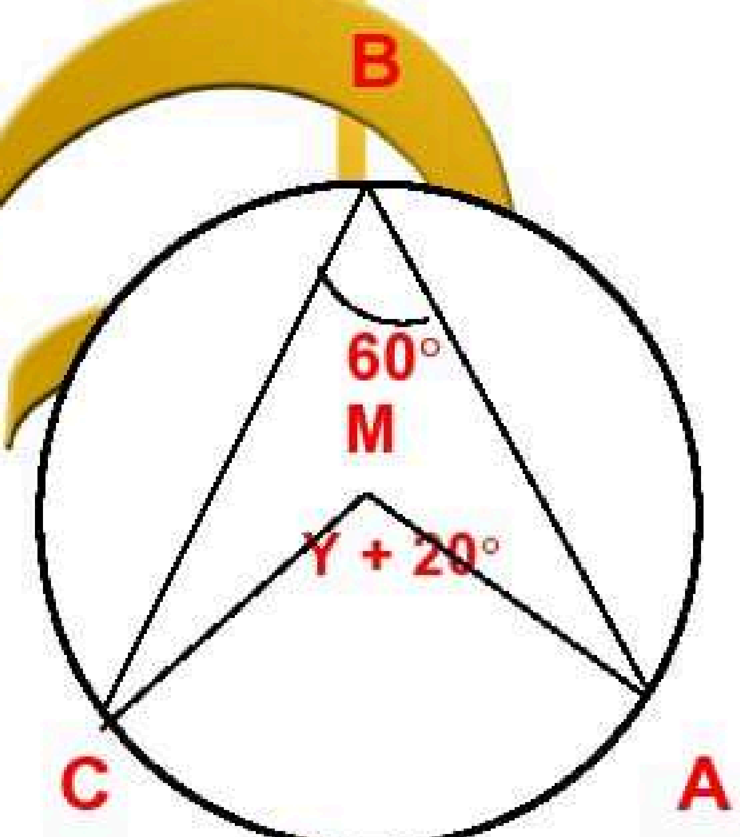
(21) A circle whose circumference 20 π cm. its area = …….. π cm^2

(a) 10 (b) 100 (c) 200 (d) 400

(22) In the opposite figure:

m (∠ ABC) = 60° , m (∠ AMC) = (y + 20°)

then y = …….°



(a) 30 (b) 40 (c) 80 (d) 100

(23) Δ ABC is a right-angled triangle at C, then the two angles A, B are ……………….

(a) supplementary (b) complementary

(c) adjacent (d) vertically opposite angles

(24) If the point A belongs to the circle M of diameter 6 cm, then MA equals …………..

(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

(25) If the circle M ⋂ the circle N= {A , B} , then the two circles M and N are …………..

(a) intersecting (b) concentric

(c) touching externally (d) distant

(26) A chord of length 8 cm, in a circle with diameter of length 10 cm, then the chord at distance from its center equals ………..

(a) 2 cm (b) 4 cm (c) 3 cm (d) 6 cm

(27) The medians of triangle intersects at a same point which each in the ratio ……………… from its base

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 2

(28) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals …………..

(a) 60° (b) 120° (c) 90° (d) 240°

(29) The chord which pass through the center of the circle is called ………….. to the circle

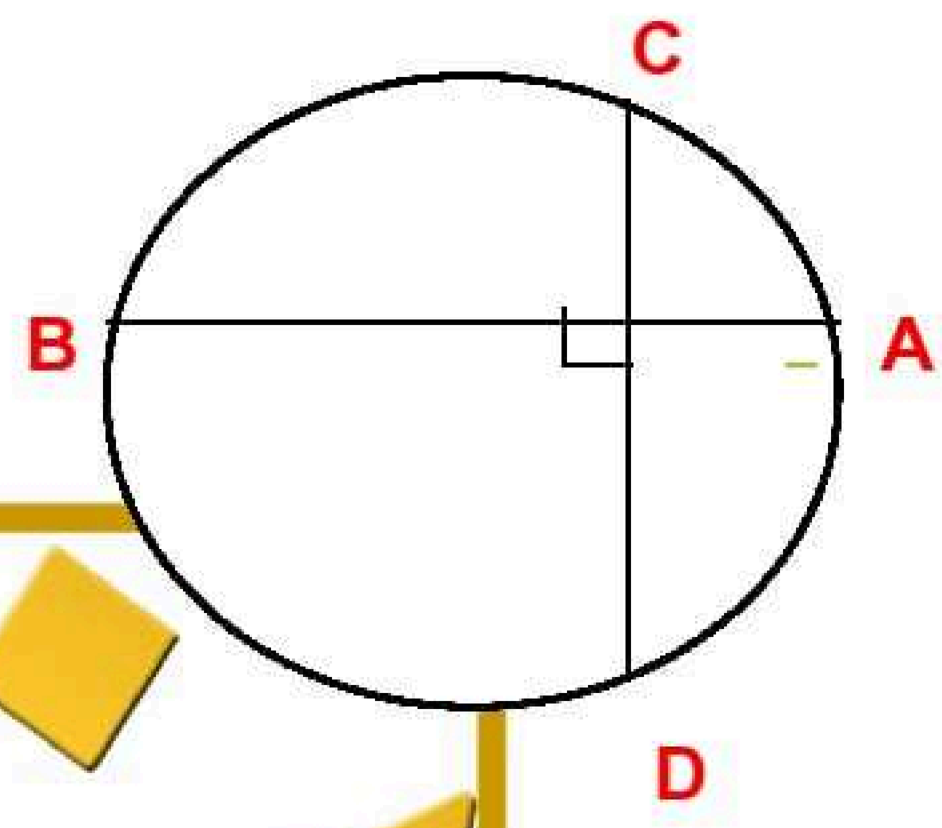
(a) tangent (b) secant (c) diameter (d) radius

(30) If m_1 , m_2 are two slopes of two parallel straight lines, then ……

(a) $m_1 + m_2 = 0$ (b) $m_1 = m_2$

(c) $m_1 \times m_2 = -1$ (d) $m_1 - m_2 = -1$

(31) In the opposite figure:



AB ⊥ CD , then m $\overset{\frown}{(AC)}$ + m $\overset{\frown}{(BD)}$ = ...°

(a) 45 (b) 90 (c) 180 (d) 270

(32) If the side lengthy of a rhombus is L cm , then its perimeter = ……… cm

(a) L^2 (b) $2L^2$ (c) $4L$ (d) $2\sqrt{2L}$

(33) The number of circles that pass through three collinear points equals …………….

(a) zero (b) 1 (c) 3 (d) an infinite number

(34) A square of perimeter 20cm, then its area = ………… cm^2

(a) 20 (b) 25 (c) 50 (d) 100

(35) If the length of an arc of a circle is $\frac{1}{3}\pi r$, then its opposite central angle of measure ………..°

(a) 30 (b) 60 (c) 120 (d) 240

(36) If cos $2x = \frac{1}{2}$ where x is an a cute angle , then m ($\angle x$) = ……°

(a) 15 (b) 30 (c) 45 (d) 60

(37) The diagonals are equal in length and not perpendicular in …….

(a) square (b) rhombus (c) rectangle (d) parallelogram

(38) The numbers 5 , 4 , ………. Can be side lengths of a triangle

(a) 8 (b) 9 (c) 10 (d) 12

(39) It is possible to draw a circle passing through the vertices of a ……………

(a) rhombus (b) square (c) trapezium (d) parallelogram

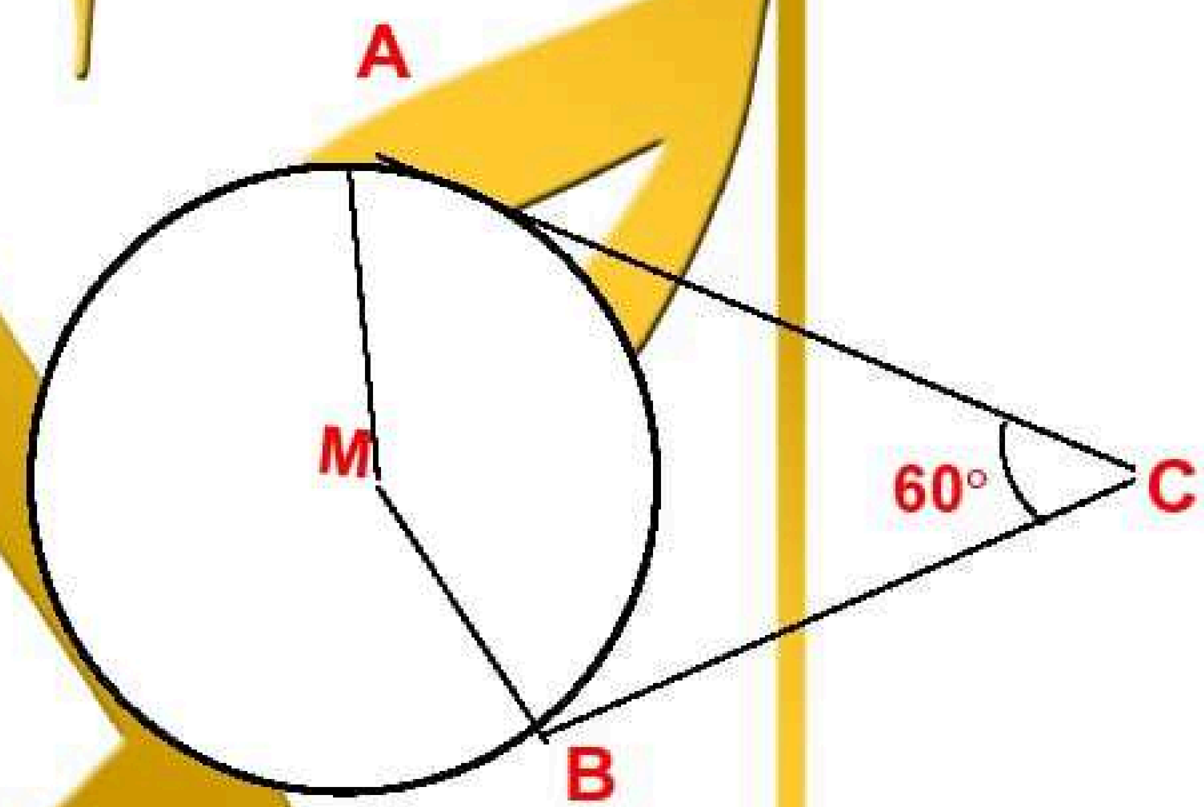
(40) Δ XYZ is right-angled triangle at Y, then XZ ………. YZ

(a) < (b) > (c) = (d) twice

(41) In the opposite figure:

CA , CB are two tangents to the Circle M , m (∠ C) = 60° ,

then m (∠ M) = …….. °



(a) 90 (b) 100 (c) 110 (d) 120

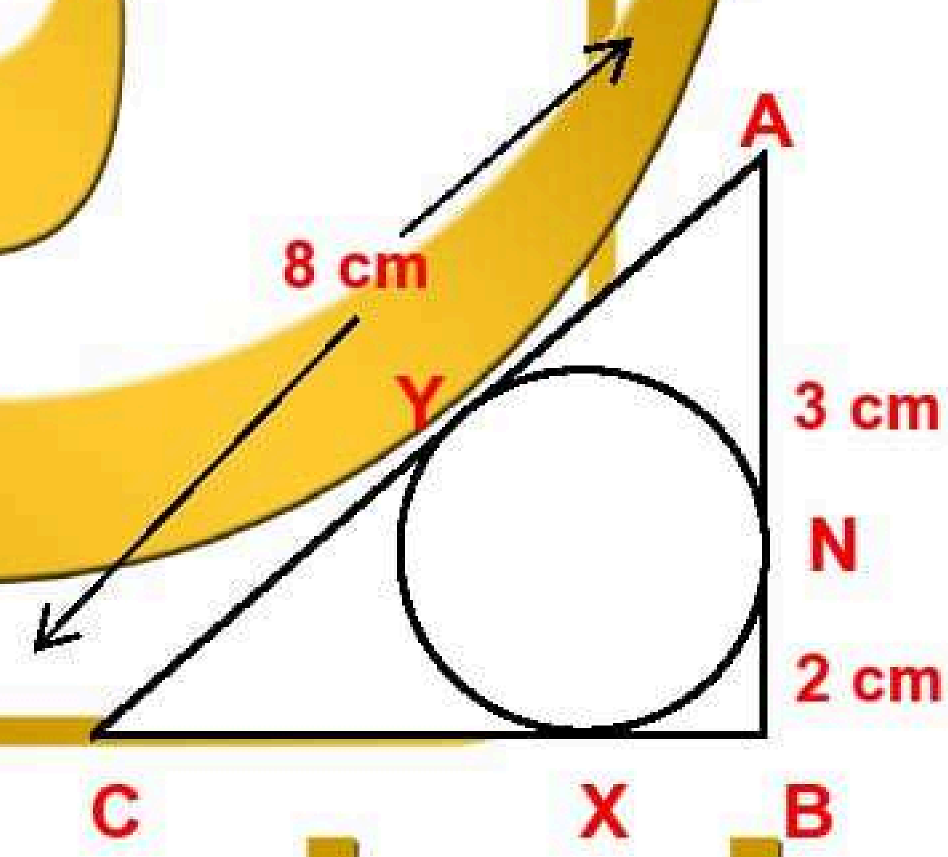
(42) M and N are two intersecting circles their radii are 5 cm, 2 cm , then MN ∈ ……………

(a)] 3 , 7 [(b) [3 , 7] (c) [3 , 7 [(d)] 3 , 7]

(43) In the opposite figure:

if AC = 8 cm , A Z = 3 cm, B Z = 2 cm,

then BC = ………..



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(a) 5 cm (b) 7 cm (c) 10 cm (d) 13 cm

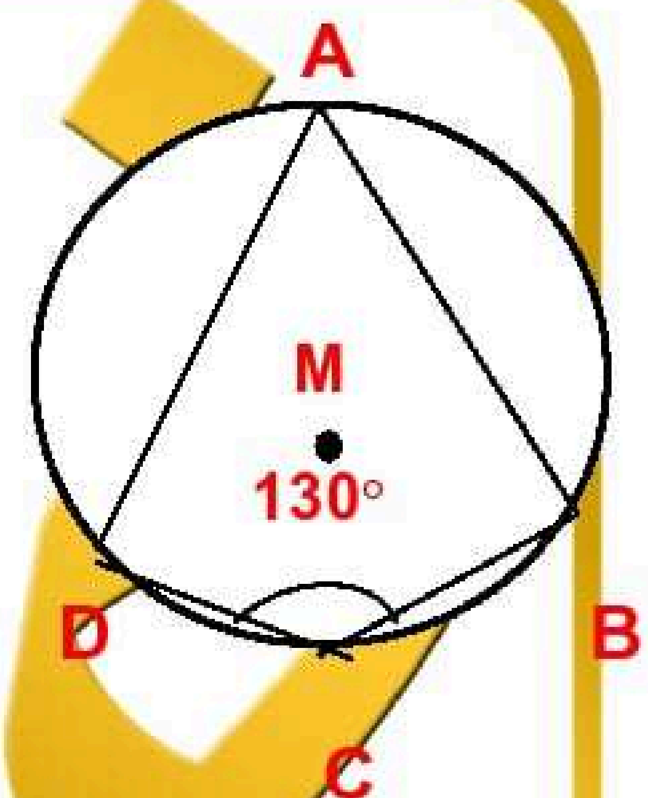
(44) The number of sides of the regular polygon in which the measure one of its interior angles 135° = ………………… sides

(a) 4 (b) 6 (c) 8 (d) 10

(45) In the opposite figure;

If M is a circle, m (∠ BCD) = 130°

Then m (∠ BAD) = ……….°



(a) 50 (b) 130 (c) 65 (d) 260

(46) The rhombus in which the lengths of its diagonals are L_1 and L_2 , its area = ……………..

(a) $L_1 L_2$ (b) $L_1 + L_2$ (c) $2\, L_1 L_2$ (d) $\frac{1}{2} L_1 L_2$

(47) The image of the point (A , B) by rotation R (0 , 180°) the point ….

(a) (− A , B) (b) (− A , − B) (c) (A , − B) (d) (A , B)

(48) The inscribed angle which opposite to the minor arc in a circle is ………..

(a) reflex (b) right (c) obtuse (d) acute

(49) If two chords intersect at a point inside a circle then the measure of the included angle equals ………. Of the two opposite arcs

(a) half of the difference (b) half of the sum

(c) twice the sum (d) twice the difference

(50) The radius length of the circle whose center is (7 , 4) and pass through the point (3 , 1) equals ……… length unit

(a) 3 (b) 4 (c) 5 (d) 6

(51) Numbers of circles passing through a given point ……..

(a) one circle (b) two circles

(c) three circles (d) infinite number of circles

(52) If the radius length of the circle M equals 2 cm, then its circumference equals ……………….

(a) 4π cm (b) 5π cm (c) 6π cm (d) 7π cm

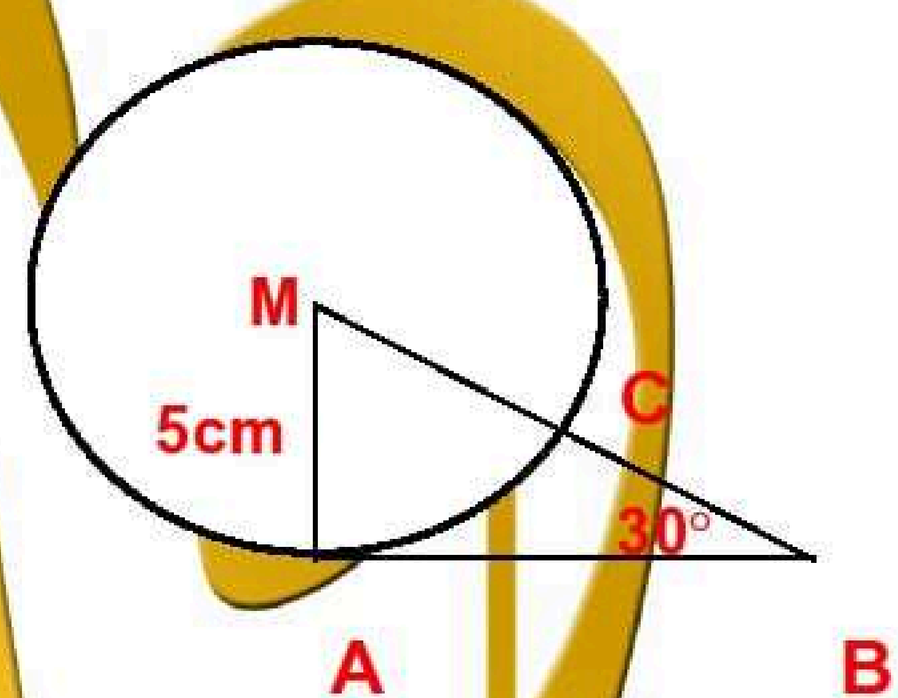
(53) If m (∠ A) = $\frac{1}{2}$ m (∠ C) in a cyclic quadrilateral ABCD, then m (∠ A) = …………°

(a) 20 (b) 30 (c) 60 (d) 120

(54) In the opposite figure:

$\overline{AB}$ is a tangent, AM = 5 cm, m (∠ B) = 30°

then the length of $\overline{BC}$ equals ……… cm



(a) 5 (b) 7 (c) 8 (d) 10

(55) The number of symmetric axes of the square is ………….

(a) 1 (b) 2 (c) 3 (d) 4

Answer the questions:

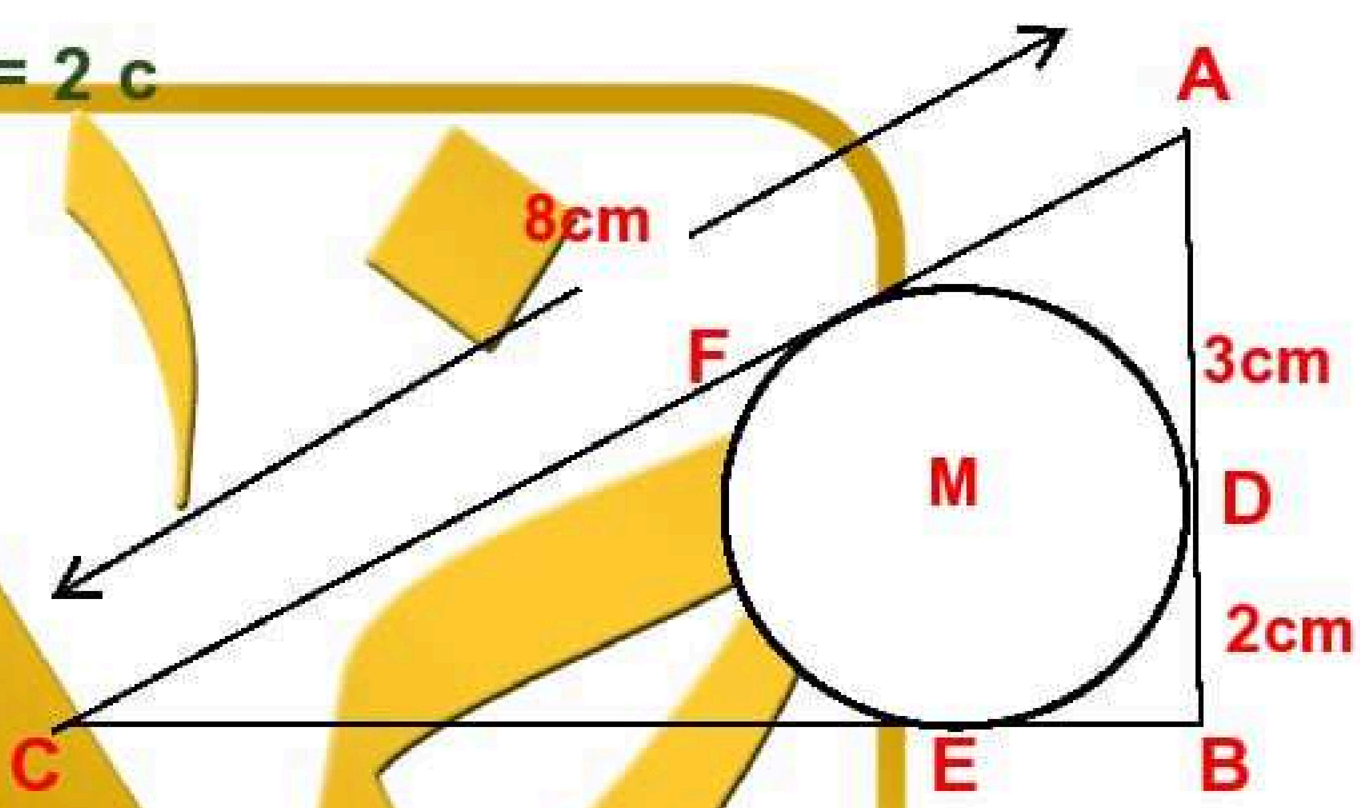
(1) In the opposite figure:

M is an inscribed circle in the triangle ABC

And touches its sides at D , E and F

AC = 8 cm , AD = 3 cm and BD = 2 c

Find: the length of $\overline{BC}$



(2) In the opposite figure:

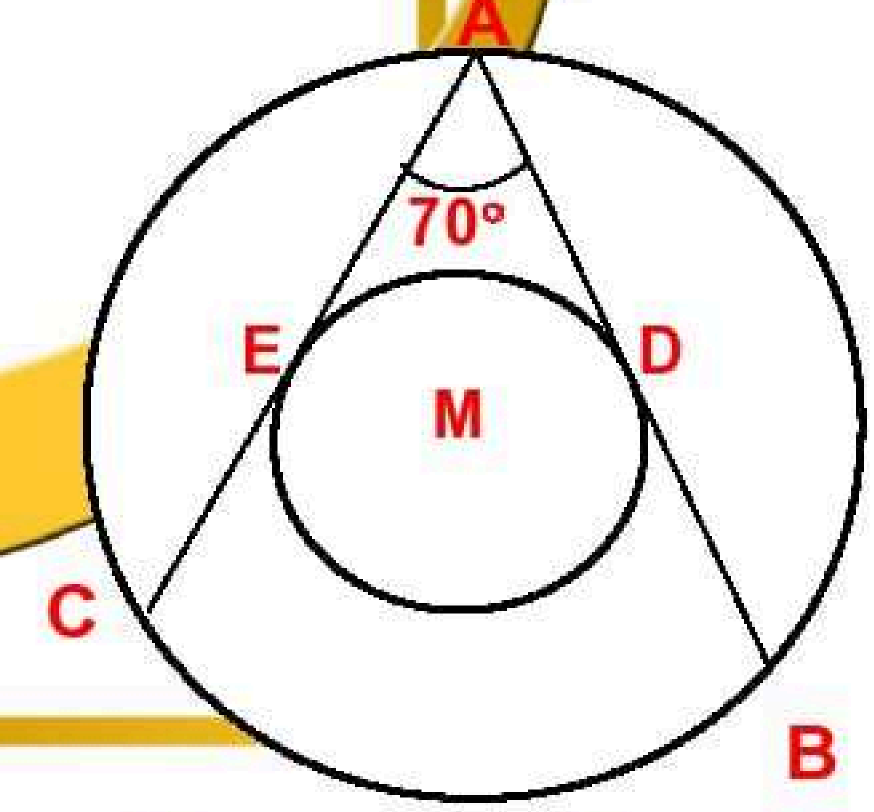
Two concentric circles at M , AB and AC

are two tangent –segment to the smallest circle

at D and E respectively and m (∠ A) = 70°

Find: (1) m (∠ DME)

(2). Prove that: AB = AC



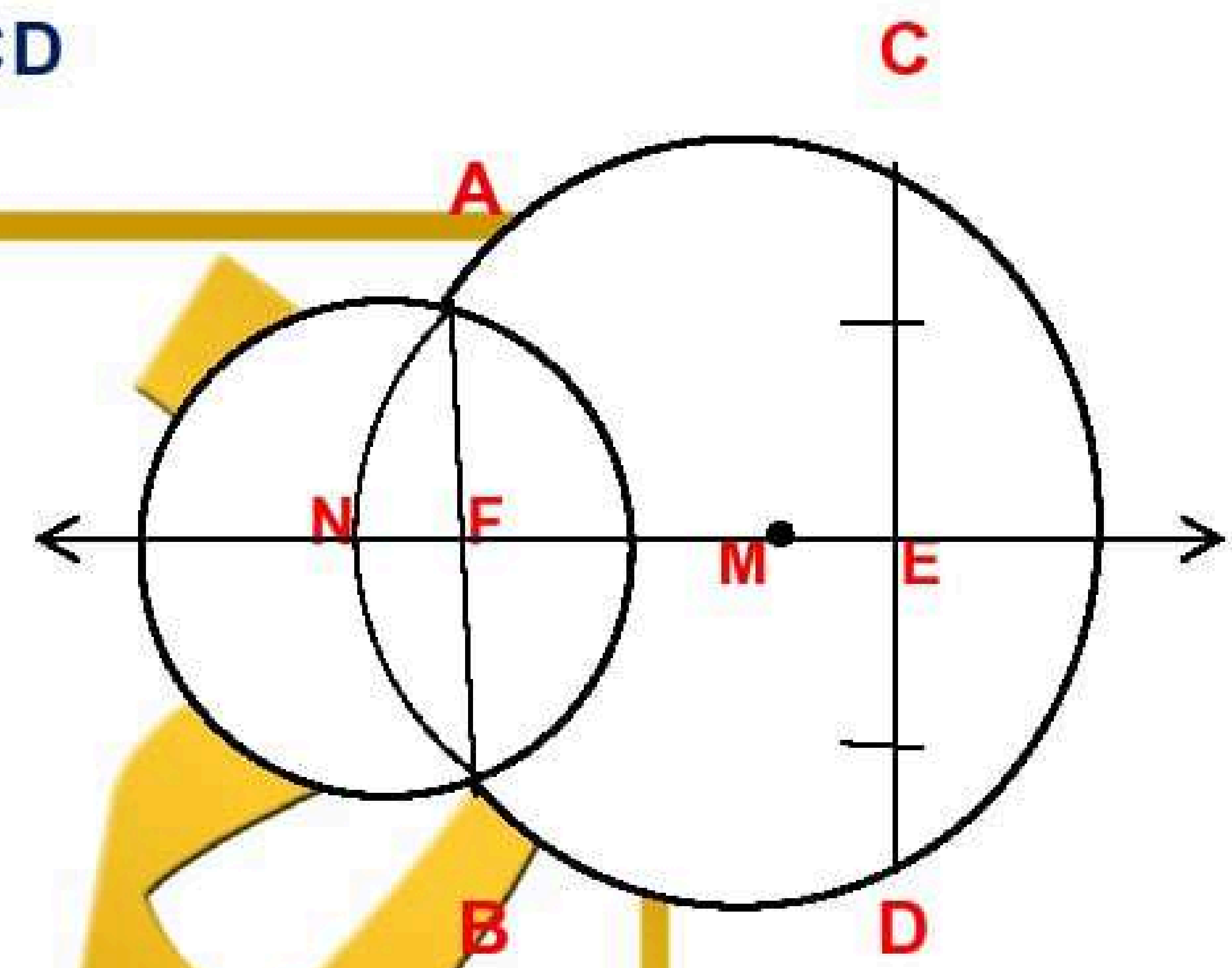
(3) In the opposite figure:

M, N are two intersecting circles

$\overline{CD}$ is a chord in the circle M

Cuts MN at E, If E is the midpoint of CD

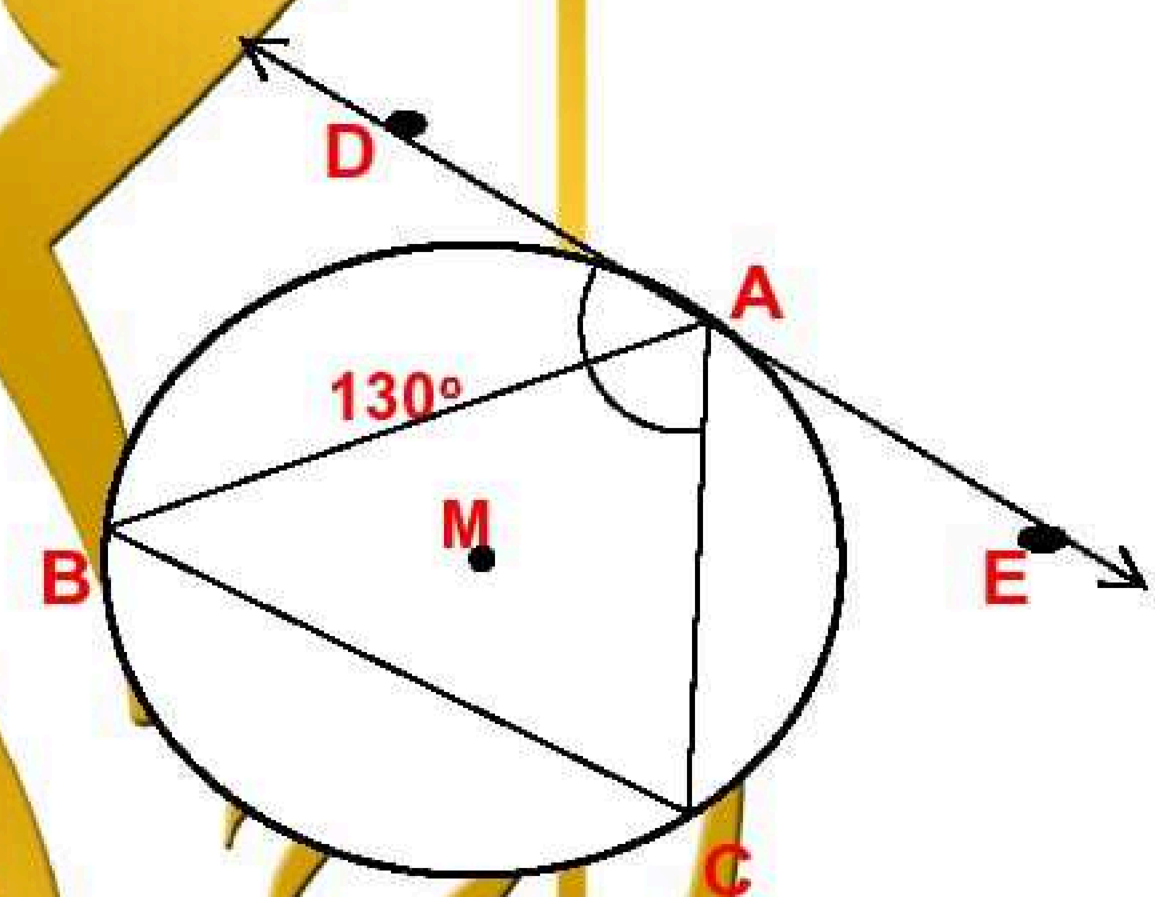
Prove that: $\overline{AB} // \overline{CD}$



(4) In the opposite figure:

$\overline{AD}$ is a tangent touches the circle M at A ,m (∠ DAC) = 130°

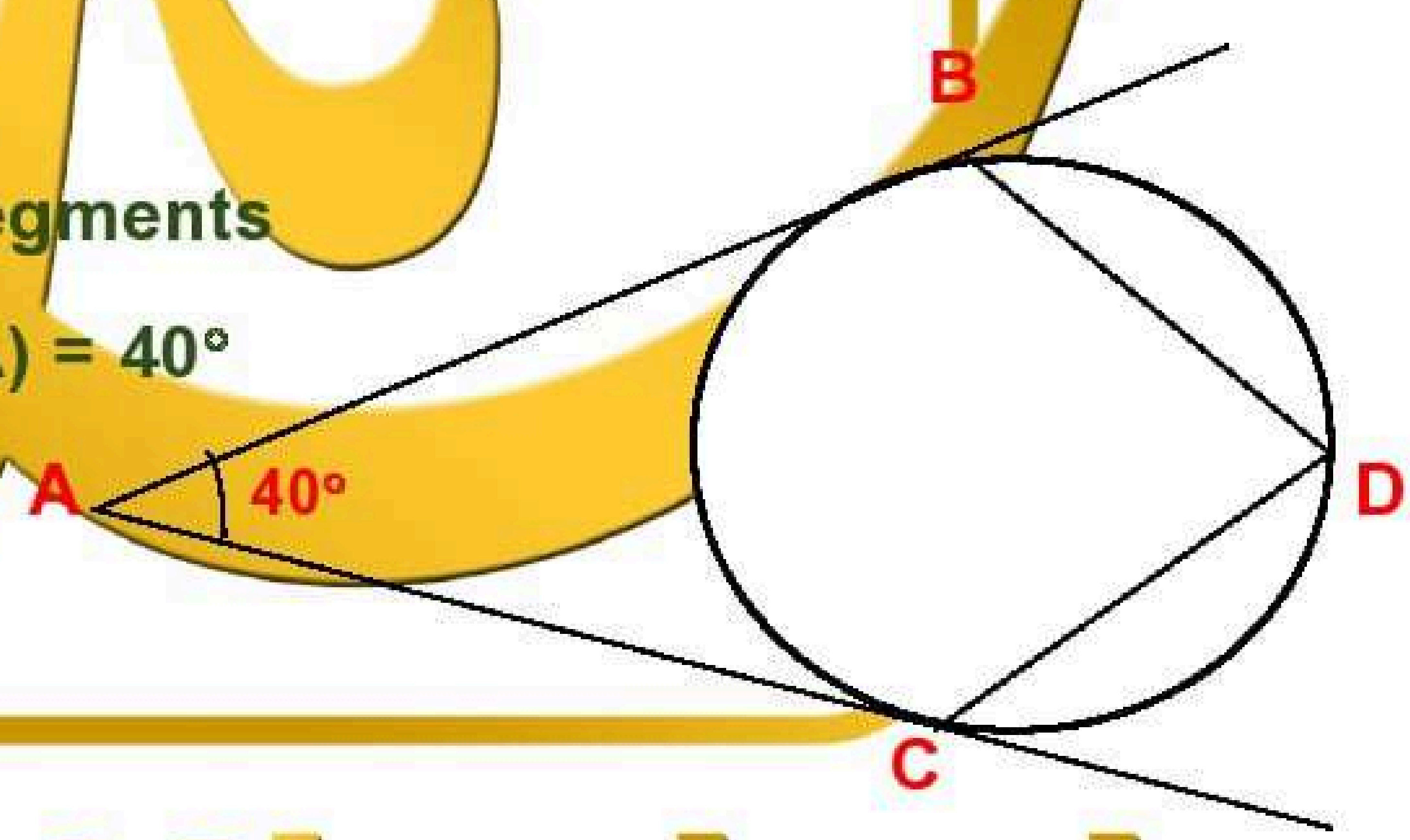
Find with proof: m (∠ B)



(5) In the opposite figure:

$\overline{AB}$, $\overline{AC}$ are two tangent –segments

To the circle at B , C , m (∠ A) = 40°



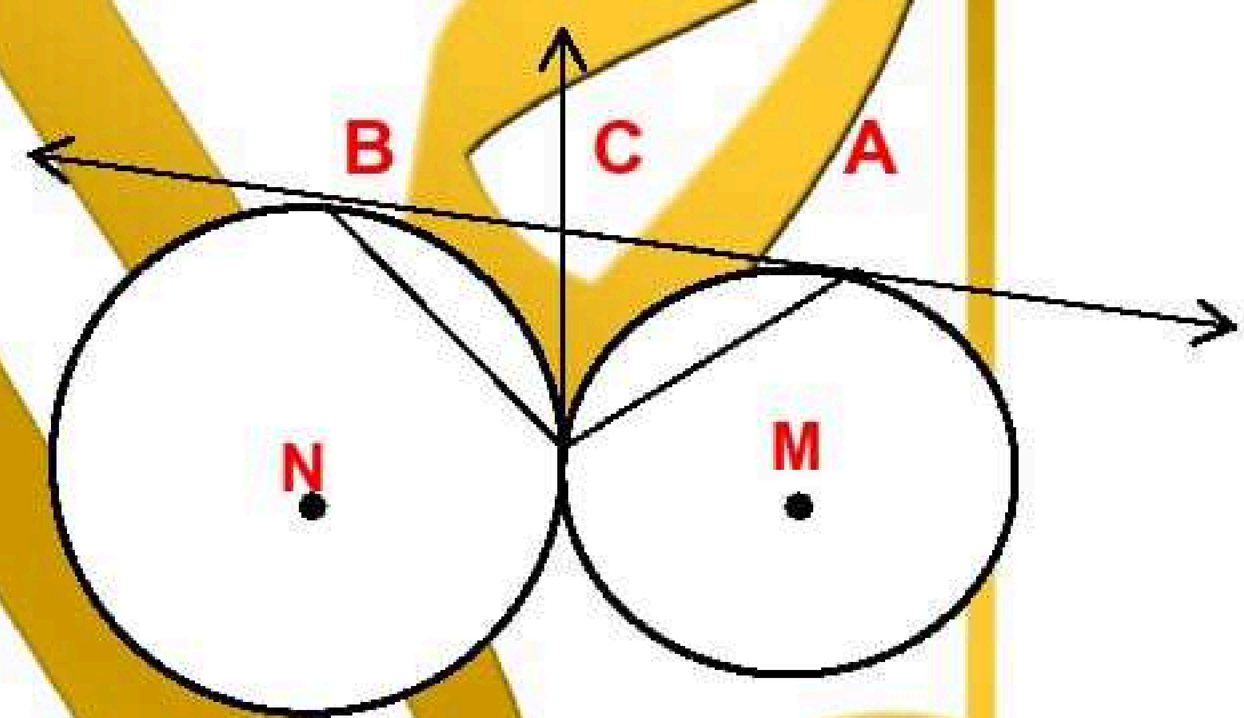
Find: m (∠ BDC)

(6) **In the opposite figure:**

M and N are two circles touching externally at D and $\overleftrightarrow{AB}$ is a common tangent to the two circles at D where $\overrightarrow{DC} \cap \overrightarrow{AB} = \{C\}$

Prove that: (1) C is the midpoint of $\overline{AB}$

(2) $\overline{AD} \perp \overline{BD}$

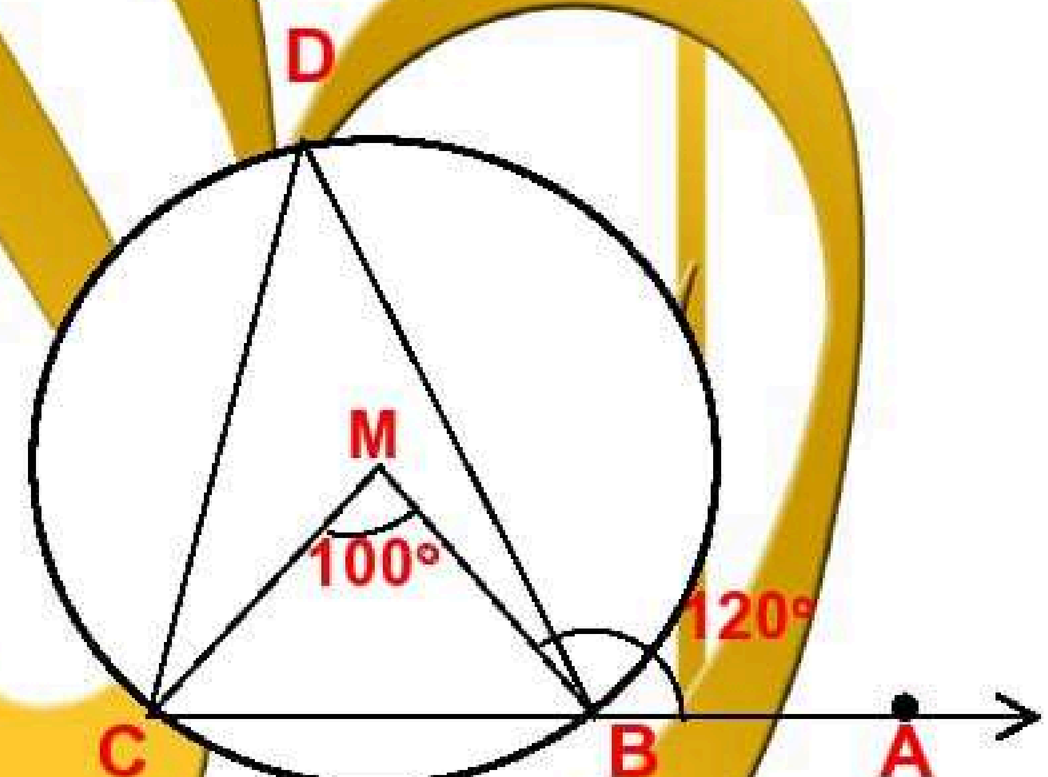


(7) **In the opposite figure:**

m (∠ BMC) = 100°

m (∠ ABD) = 120°

Find with proof: m (∠ DCB)



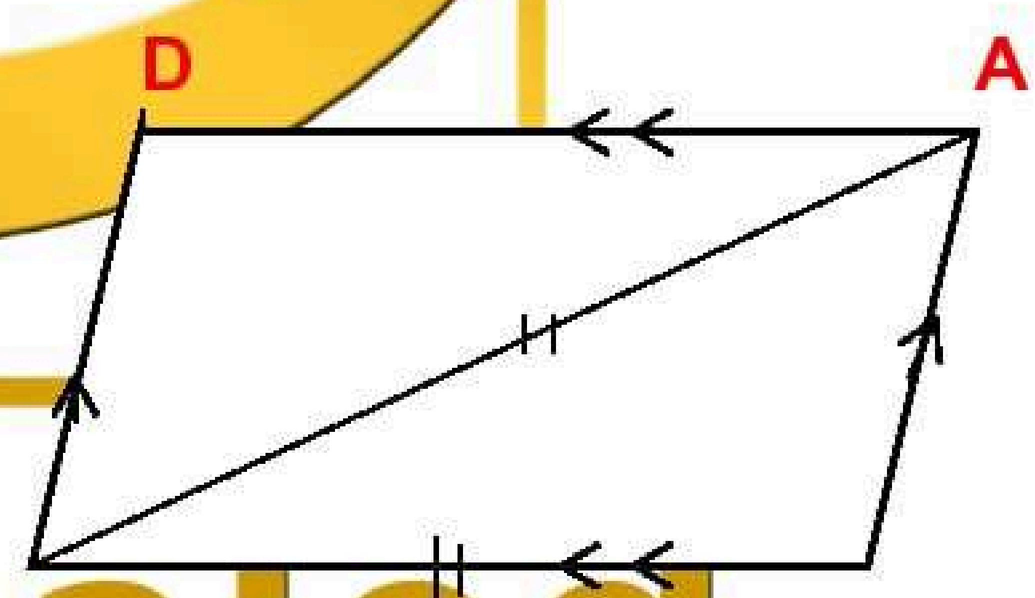
(8) **In the opposite figure:**

ABCD is a parallelogram in which

AC = BC

Prove that:

$\overleftrightarrow{CD}$ is a tangent to the circumcircle of



The triangle ABC C B

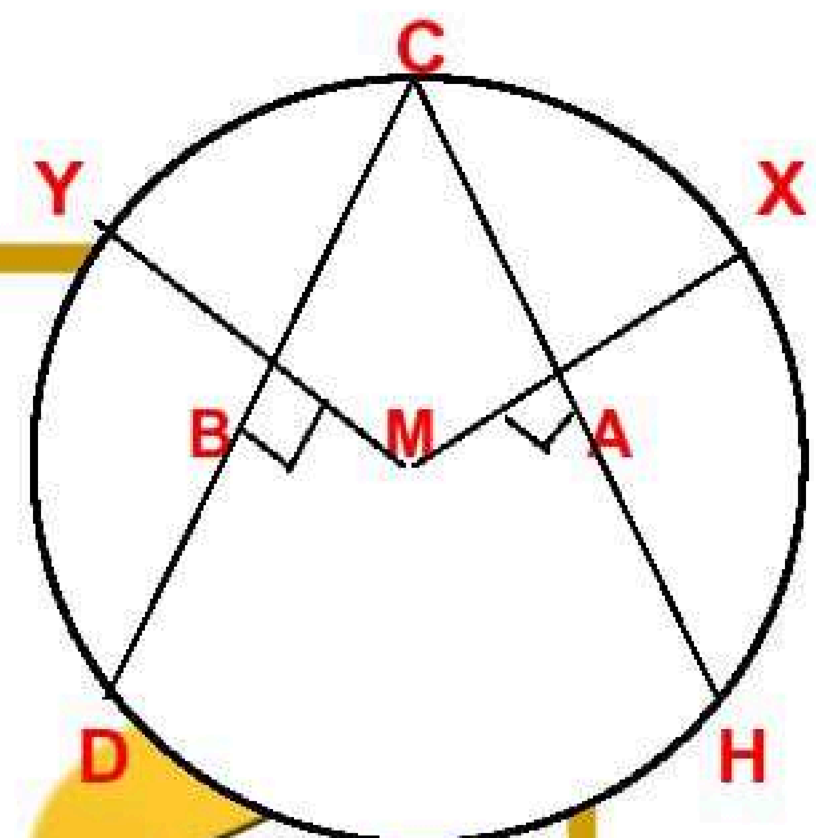
(9) In the opposite figure:

A circle of center M,

CD = CH , $\overline{MX} \perp \overline{CH}$,

$\overline{MY} \perp \overline{CD}$

Prove that: AX = BY

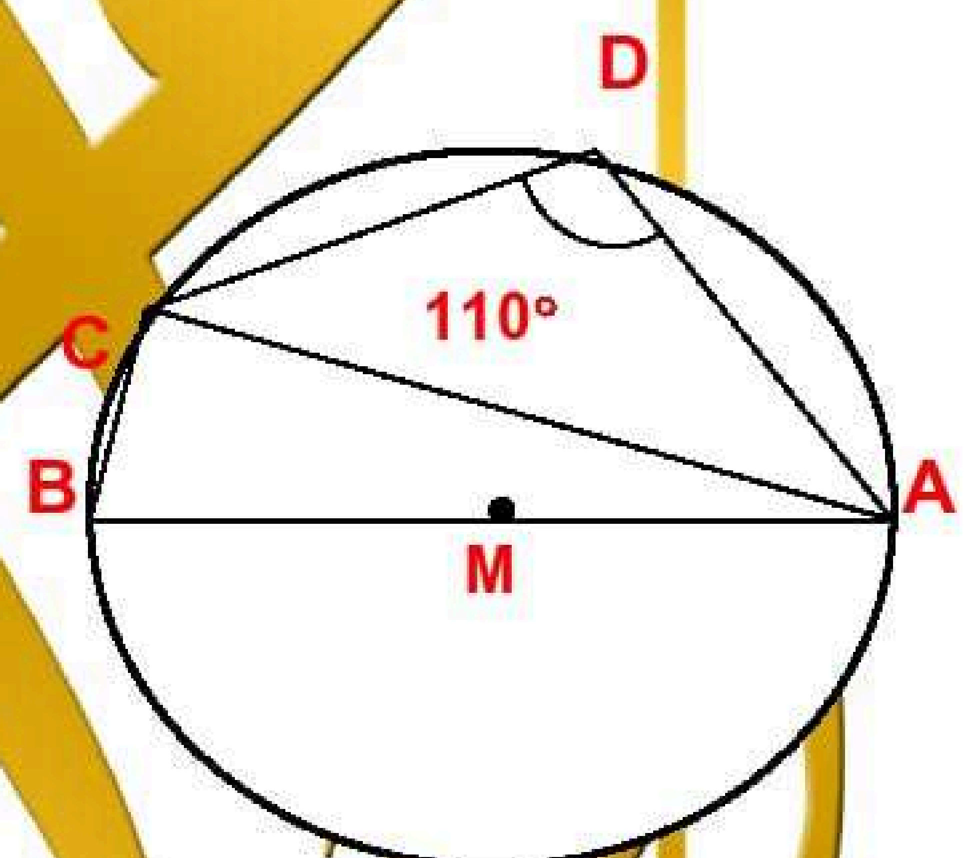


(10) In the opposite figure:

$\overline{AB}$ is a diameter of a circle M

m (∠ CAD) = 110°

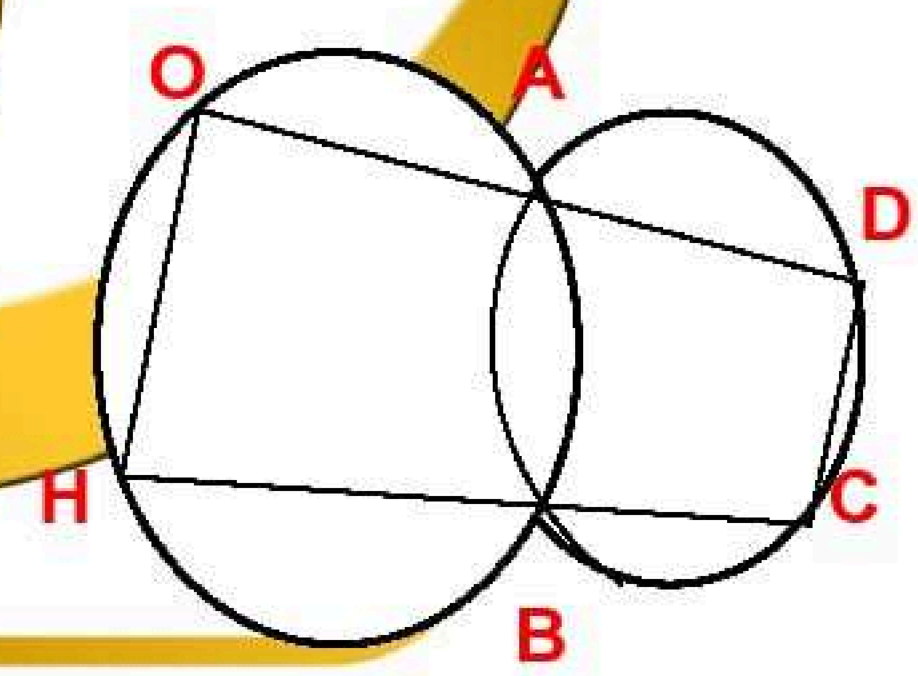
Find: m $\overset{\frown}{(CB)}$



(11) In the opposite figure:

Two circles are intersecting at A , B

Prove that: $\overline{DC}$ // $\overline{OH}$



(12) ABCD is a parallelogram in which AC= BC

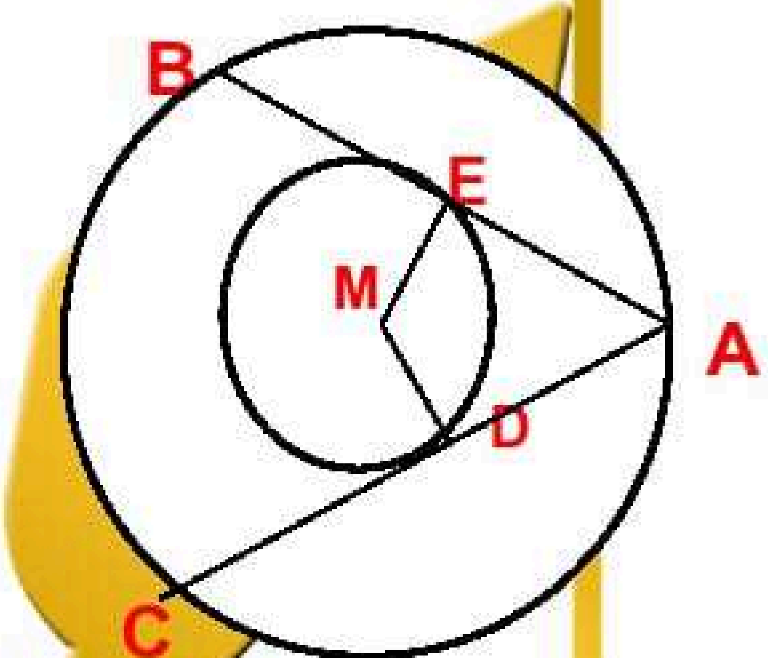
Prove that: CD is a tangent to the circle passing through the vertices of the triangle ABC

(13) In the opposite figure:

Two concentric circles at M

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle

Prove that: AB = AC



(14) In the opposite figure;

AB is a diameter of a circle M ,

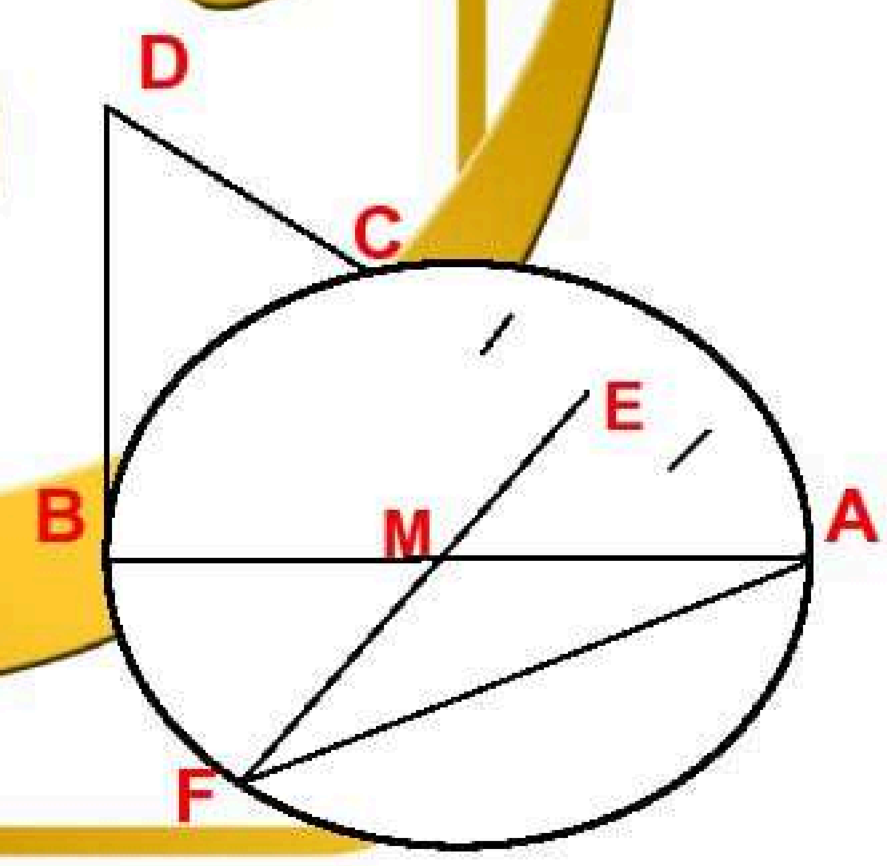
E is the midpoint of a chord $\overline{AC}$,

$\overrightarrow{BD}$ is a tangent to the circle at B,

Where $\overrightarrow{BD} \cap \overrightarrow{AC}$ = {D} and $\overrightarrow{EM}$ is drawn to cut the circle at F

Proof that: (1) MEDB is a cyclic quadrilateral

(2) m (∠ F) = $\frac{1}{2}$ m (∠ D)



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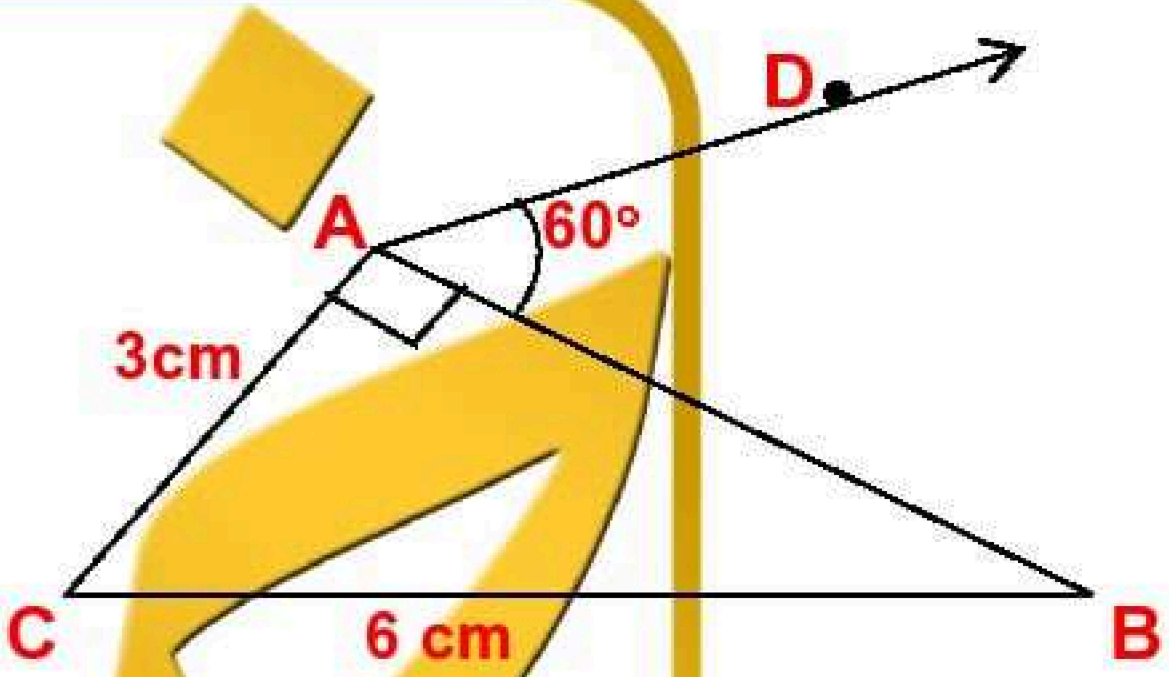
(15) In the opposite figure:

ABC is a right-angled triangle at A,

AC = 3 cm , BC = 6 cm,

m (∠ DAB) = 60°



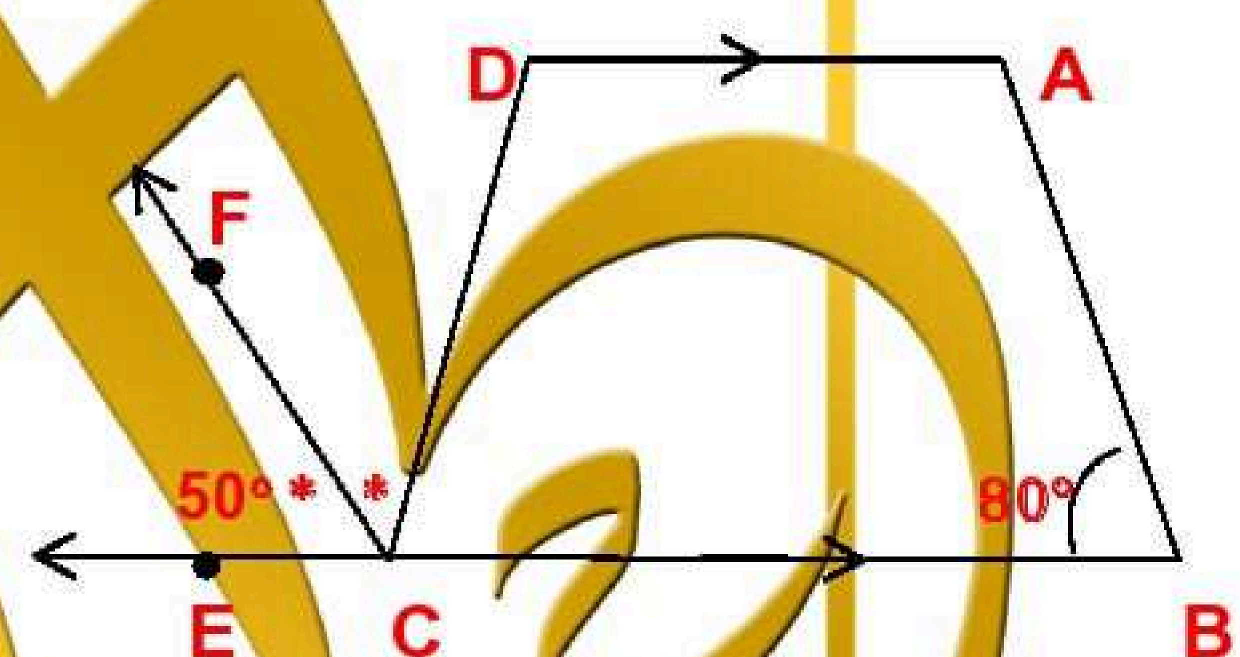
Prove that: $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC

In the opposite figure:

$\overline{AD}$ // $\overrightarrow{BC}$, m (∠ B) = 80° ,

$\overrightarrow{CF}$ is bisects ∠ DCE

, m (∠ FCE) = 50°



Prove that: the figure ABCD is a cyclic quadrilateral

(16) In the opposite figure:

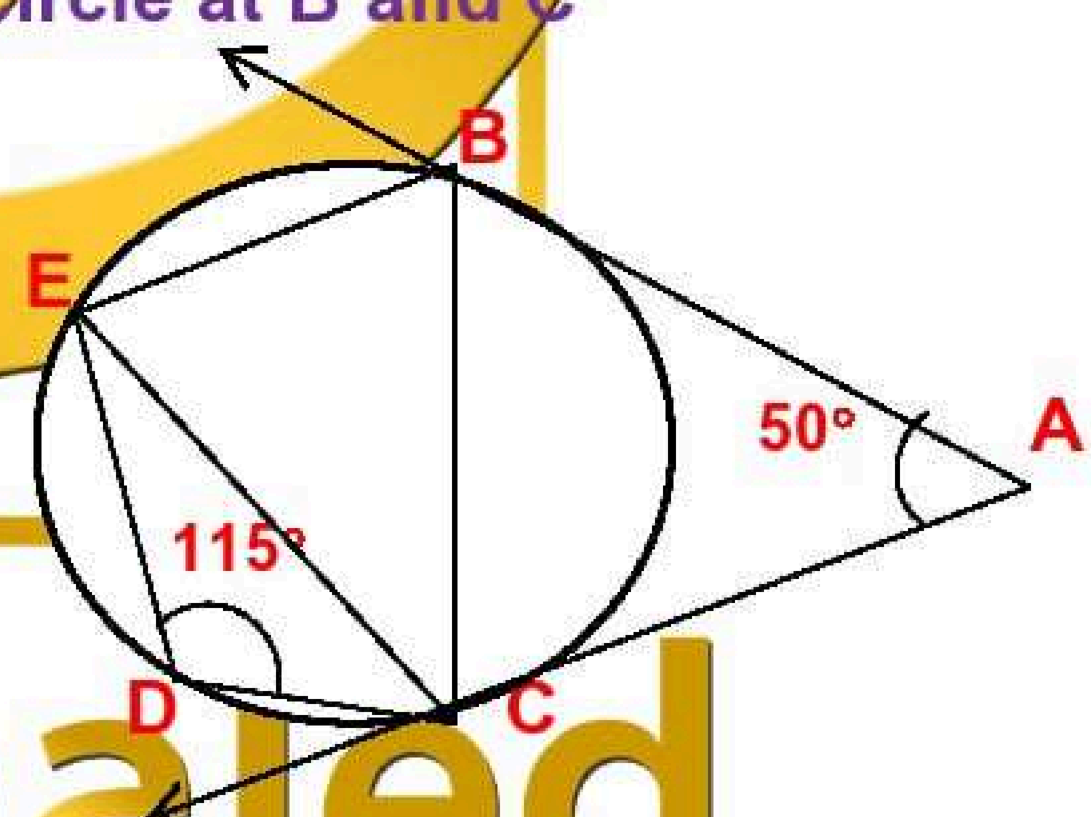
$\overline{AB}$, $\overline{AC}$ are two tangent segment to the circle at B and C

, m (∠ A) = 50°

, m (∠ CDE) = 115°

Prove that: (1) $\overrightarrow{BC}$ bisect ∠ ABC

(2) CB = CE



1 Two distance circles M and N with radii lengths 6 cm and 8 cm respectively , then MN .... 14 cm. « Cairo 2019 »

(a) $>$ (b) $\geqslant$ (c) $<$ (d) $=$

2 The measure of inscribed angle is .... the measure of the central angle subtended by the same Arc .

(a) Half (b) Twice (c) Quarter (d) Third « Cairo 2019 »

3 In the cyclic quad , if : $m(\angle A) = \frac{1}{2} m(\angle C)$, then : $m(\angle A) =$ ....

(a) 20° (b) 30° (c) 60° (d) 120° « Cairo 2019 »

4 The measure of inscribed angle in a semicircle = ... « Giza , S.sinai 2019 »

(a) 45° (b) 90° (c) 120° (d) 180°

5 Two circles M and N touching internally , their radii lengths 3 cm and 5 cm respectively , then MN = ... cm. « Giza 2019 »

(a) 3 (b) 5 (c) 2 (d) 8

6 If the surface of circle M ∩ the surface of the circle N = { A } and the radius length of one of them equals 3 cm. , and MN = 8 cm , then the radius length of the other circle = ... cm. « Alex 2019 »

(a) 5 (b) 6 (c) 11 (d) 16

7 A circle can be drawn passing through the vertices of a .... « Alex 2019 , sharkia 2019 »

(a) Rhombus (b) Parallelogram (c) Trapezium (d) Rectangle

8 A circle with diameter length equals 10 cm. , the straight line L is distant from its center by 5 cm. , then the straight line L is .... « sharkia 2019 »

(a) a tangent (b) a secant
(c) Outside the circle (d) a diameter of the circle

9 The number of common tangents of two touching circles externally equals .... « sharkia 2019 »

(a) zero (b) 1 (c) 2 (d) 3

10 If M and N are two touching circles externally , the lengths of their radii are 2 cm. , and 4 cm. Respectively , then the area of the circle with diameter $\overline{MN}$ equals .... cm^2. « sharkia 2019 »

(a) 36π (b) 9π (c) 16π (d) 4π

11 M and N are two circles , whose radii lengths are 6 cm. , and 8 cm. and MN = 14 cm. Then the two circles are .... « Dakahlia 2019 »

(a) Distant (b) Intersecting
(c) One inside the other (d) Touching externally

12 A circle with greatest chord with length = 12 cm. , then the circumference of the circle = ...
« Dakahlia 2019 »

ⓐ 12π ⓑ 24π ⓒ 6π ⓓ 10π

13 The inscribed angle drawn in a semicircle is .... « Dakahlia , r.sea 2019 »

ⓐ an acute ⓑ a straight ⓒ an obtuse ⓓ a right

14 A chord is of length 8 cm. in a circle of diameter length 10 cm.
, then the chord is at .... from the center of the circle. « Dakahlia 2019 »

ⓐ 2 cm. ⓑ 3 cm. ⓒ 4 cm. ⓓ 6 cm.

15 The number of common tangents of two touching circles internally is .... « Dakahlia 2019 »

ⓐ zero ⓑ 1 ⓒ 2 ⓓ 3

16 In the cyclic quad , if : $m(\angle A) + 3\ m(\angle C) = 280^\circ$, then : $m(\angle C) =$ ....

ⓐ 50° ⓑ 150° ⓒ 130° ⓓ 100°

17 the measure of the central angle drawn in $\frac{1}{3}$ circle equals .... « ismailia 2019 »

ⓐ 240° ⓑ 120° ⓒ 60° ⓓ 30°

18 Which of the following figures is cyclic quadrilateral .... « ismailia 2019 »

ⓐ the Rhombus ⓑ the Parallelogram ⓒ the Trapezium ⓓ the Rectangle

19 If AB = 8 cm. then the radius length of the smallest circle can be drawn passing through the two points A and B equals .... cm. « ismailia 2019 »

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4

20 If M and N are two intersecting circles whose radii length are 5 cm. and 2 cm.
, Then : $MN \in$ .... « kalyoubia 2019 »

ⓐ $]3,7[$ ⓑ $[3,7]$ ⓒ $[3,7[$ ⓓ $]3,7]$

21 the measure of the central angle which is opposite to an arc of length $\frac{1}{3}\pi r$ .... « kalyoubia 2019 »

ⓐ 30° ⓑ 60° ⓒ 120° ⓓ 240°

22 The axis of symmetry of a circle is .... « Monofia 2019 »

ⓐ The diameter ⓑ The chord
ⓒ The tangent ⓓ The straight line passing through the center

23 ABCD is a cyclic quad in which , $m(\angle A) = 2\ m(\angle C)$, then : $m(\angle A) =$ .... « Dakahlia , monofia 2019 »

ⓐ 30° ⓑ 60° ⓒ 90° ⓓ 120°

24 The longest chord in the circle is called a .... « p.said 2019 »

ⓐ diameter ⓑ Tangent ⓒ chord ⓓ radius

25 If the two circles M and N touching externally and the radius length of one of them equals 3 cm. , and MN = 8 cm , then the radius length of the other circle = ... cm. « Suez 2019 »

ⓐ 5 ⓑ 6 ⓒ 11 ⓓ 16

26 If the straight line L is a tangent to the circle M of diameter length equals 10 cm. , then the distance between L and the center of the circle equals .... cm. « p.said 2019 »

ⓐ 3 ⓑ 4 ⓒ 5 ⓓ 10

27 The ratio between The measure of the inscribed angle and the measure of the central angle subtended by the same Arc is .... « matrouh 2019 »

ⓐ 1 : 2 ⓑ 2 : 1 ⓒ 3 : 2 ⓓ 2 : 3

28 A chord with length 8 cm. in a circle with circumference 10π cm. , then it is distant from its center by............... cm. « matrouh 2019 »

ⓐ 2 ⓑ 3 ⓒ 4 ⓓ 5

29 The angle of tangency is included between .... « r.sea 2019 »

ⓐ Two chords ⓑ Two tangents

ⓒ a chord and a tangent ⓓ a chord and a diameter

30 The number of symmetry axis of the semicircle is .... « r.sea 2019 »

ⓐ 0 ⓑ 1 ⓒ 3 ⓓ an infinite number

31 The number of symmetry axis of the circle is .... « S.sinai , sohag 2019 »

ⓐ 0 ⓑ 1 ⓒ 3 ⓓ an infinite number

32 the diameter length of the circle whose center is the origin point and passes through the point (3 , – 4) equals .... length unit.

ⓐ 2.5 ⓑ 5 ⓒ 10 ⓓ 20

33 If the surface of circle M ∩ the surface of the circle N = { A }, then M and N are .... « N.sinai 2019 »

ⓐ distant ⓑ Concentric

ⓒ Touching externally ⓓ Intersecting

34 ABCD is a cyclic quadrilateral , then : $m(\angle A) + m(\angle C) - 80^\circ =$ .... « Aswan 2019 »

ⓐ 60° ⓑ 80° ⓒ 100° ⓓ 120°

35 The length of the arc subtending a central angle of measure 60° in a circle whose circumference Is 24 cm. equals .... cm. « Luxor 2019 »

ⓐ 4 ⓑ 8 ⓒ 12 ⓓ 16

36 If A , B two points in the plane , AB = 7 cm. then the diameter length of the smallest circle passing through the two points A and B equals .... cm. « Qena 2019 »

ⓐ 3 ⓑ 3.5 ⓒ 7 ⓓ 14

37 The diameter is a .... passing through the center of the circle. « Assiut 2019 »

ⓐ ray ⓑ Straight line ⓒ tangent ⓓ chord

38 If the circumference of a circle is 20π cm. , then its area = ... cm^2.

ⓐ 10 ⓑ 20 ⓒ 100π ⓓ 400π

39 The symmetry axis of the common chord $\overline{AB}$ of the two intersecting circles M , N is .... « B.suef 2019 »

ⓐ $\overleftrightarrow{MA}$ ⓑ $\overleftrightarrow{MN}$ ⓒ $\overleftrightarrow{MB}$ ⓓ $\overleftrightarrow{AB}$

40 If M is circle of diameter length 8 cm. , the straight line L is far from the center M of the circle 4 cm. , then the straight line L is .... « Fayoum 2019 »

ⓐ a secant to the circle in two points. ⓑ Outside the circle.
ⓒ A tangent to the circle. ⓓ an axis of symmetry of the circle.

41 the center of the circle that passing through the vertices of the triangle is the intersection point of .... « Fayoum 2019 »

ⓐ The bisectors of its interior angles. ⓑ The bisectors of its exterior angles.
ⓒ Its altitudes. ⓓ The axis of its sides.

42 If M is circle of diameter length 8 cm. , the straight line L is far from the center M of the circle 4 cm. , then the straight line L is .... « Fayoum 2019 »

43 If the straight line L is a tangent to the circle M of diameter length equals 8 cm. , then L is at a distance of .... cm. from the centre. « kalyoubia 2018 »

ⓐ 3 ⓑ 4 ⓒ 5 ⓓ 10

44 If M is circle , its diameter length = 14 cm. , MA = $(2x + 3)$ cm. where A is a point on the circle. , then : $x =$ .... « Sharkia 2015 »

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 5

45 A circle of circumference 6π cm. , and the straight line L is distance from its centre by 3 cm. , then the straight line L is .... « monofia 2015 »

ⓐ a diameter of the circle. ⓑ a secant.
ⓒ A tangent to the circle. ⓓ Outside the circle.

46 If M is circle , its diameter length = $(2x + 5)$ cm. , and the straight line L is distance $(x + 2)$ cm. from its centre circle , then the straight line L is .... « P.said 2017 »

ⓐ a secant to the circle in two points. ⓑ Outside the circle.
ⓒ A tangent to the circle. ⓓ an axis of symmetry of the circle.

47 Two circles M and N with radii lengths 4 cm. and 7 cm. respectively , are touching , Then : MN $\in$ ....

ⓐ $]3,11[$ ⓑ $[3,11]$ ⓒ $\mathbb{R}-[3,7]$ ⓓ $\{3,11\}$

48 If the radii lengths of the two circles M and N are 6 cm. and 3 cm. MN = 2 cm. circle. , then the two circles M and N are .... « Dakahlia 2018 »

ⓐ Distant ⓑ Intersecting
ⓒ One inside the other ⓓ Touching externally

49 The number of circles passing through three collinear points is .... « Giza 2016 , Souhag 2018 »

ⓐ zero ⓑ one ⓒ three ⓓ an infinite number

50 The number of circles passing through three collinear points is .... « menia 2017 »

ⓐ zero ⓑ one ⓒ two ⓓ three

51 The type of the inscribed angle which is opposite to an arc greater than the semicircle is .... « N.vally 2018 »

ⓐ acute ⓑ obtuse ⓒ straight ⓓ right

52 The centre of the circles passing through the two points A and B lies on .... « menia 2017 »

ⓐ the axis of symmetry of $\overline{AB}$ ⓑ $\overline{AB}$
ⓒ The perpendicular to $\overline{AB}$ ⓓ the midpoint of $\overline{AB}$

53 The length of the arc which represents $\frac{1}{4}$ the circumference of the circle = ... cm. « menia 2017 »

ⓐ $2\pi r$ ⓑ πr ⓒ $\frac{1}{2}\pi r$ ⓓ $4\pi r$

54 Its impossible to draw a circle passing through the vertices of a .... « B.suef 2017 »

ⓐ rectangle ⓑ triangle ⓒ square ⓓ rhombus

55 The inscribed angle which is subtended by minor arc in a circle is .... « Qena 2016 »

ⓐ acute ⓑ obtuse ⓒ straight ⓓ right

56 The number of tangents can be drawn from a point lies on a circle is .... « Beheira 2017 »

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ Infinite number

57 The number of common tangents of two intersecting circles is ....

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4

58 The number of common tangents of two distant circles is ....

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4

59 The ratio between The measure of the inscribed angle and the measure of the angle of the tangency subtended by the same Arc is .... «

ⓐ 1 : 2 ⓑ 2 : 1 ⓒ 1 : 1 ⓓ 1 : 3

60 If : $\overleftrightarrow{AB}$ ∩ the circle M = { A , B }, then : $\overleftrightarrow{AB}$ ∩ the surface of the circle M = ...

a) { A , B }
b) $\overline{AB}$
c) $\overrightarrow{AB}$
d) $\overleftrightarrow{AB}$

61 If $\overline{MA}$ and $\overline{MB}$ are two perpendicular radii in the circle M and the area of the triangle MAB = 8 cm^2.

, then the radius length of the circle = ... cm.

a) 2
b) 4
c) 8
d) 16

62 A circle of radius length = 2 cm. , then its circumference = ... cm. « Aswan 2016 »

a) 4 π
b) 5 π
c) 6 π
d) 7 π

63 the two opposite angles in the cyclic quadrilateral are .... « Alex 2017 »

a) equal
b) Supplementary
c) Complementary
d) Alternate

64 If : the circle M ∩ the circle N = { A , B }, then the two circles are .... « Ismailia 2018 »

a) Distant
b) Intersecting
c) Concentric
d) Touching

65 ABCD is a cyclic quadrilateral , in which : m (∠ A) = 75° , then : m (∠ C) = ... « R.sea 2016 »

a) 75°
b) 125°
c) 150°
d) 105°

66 Which of the following points doesn't belong to the circle whose centre is the origin and its radius length = 7 cm ? « Giza 2016 »

a) (0 , 7)
b) (0 , – 7)
c) (7 , 0)
d) (7 , 7)

67 ABCDEF is a regular hexagon drawn inside the circle M , then : m ($\overparen{BC}$) = ...

a) 30°
b) 60°
c) 90°
d) 120°

68 A circle with diameter length = (2 x) cm. , and the straight line L is distance (x + 1) cm. from its centre circle , then the straight line L will be .... « Dakahlia 2018 »

a) secant.
b) Outside
c) tangent.
d) axis of symmetry.

69 ABC is an equilateral triangle drawn inscribed in circle M , then : m ($\overparen{AB}$) = ...« Fayoum 2018 »

a) 30°
b) 60°
c) 90°
d) 120°

70 the measure of the in inscribed angle which is drawn in $\frac{1}{3}$ a circle equals .... « Dakahlia 2018 »

a) 240°
b) 120°
c) 60°
d) 30°

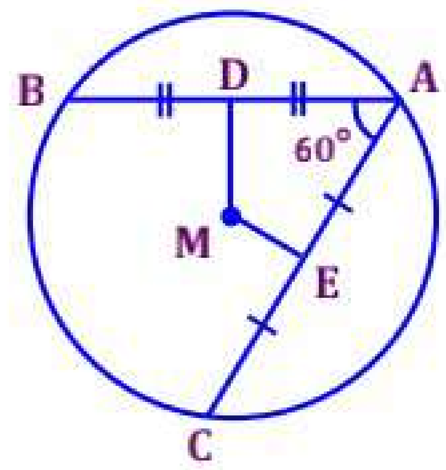
71 $\overline{AB}$ and $\overline{DC}$ are two intersected chord at the point X in the circle M , and m ($\overparen{AC}$) + m ($\overparen{BD}$) = 130°.

Then m (∠ AXC) = ...

a) 260°
b) 130°
c) 65°
d) 60°

In each of the following figures, choose the correct answer

1 in the opposite figure :

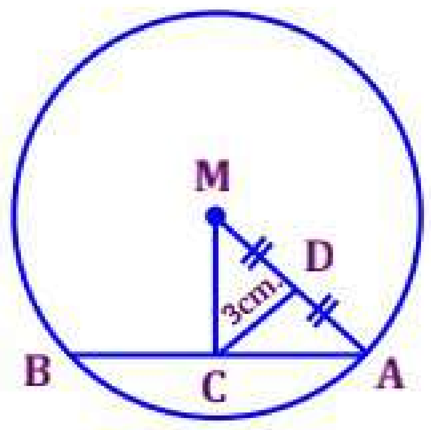


If D and E is the midpoints of $\overline{AB}$ and $\overline{AC}$, and $m(\angle BAC) = 60°$

Then : $m(\angle DME) =$ ....

(a) 60° (b) 120°
(c) 90° (d) 30°

2 in the opposite figure : « Gharbia 2019 »

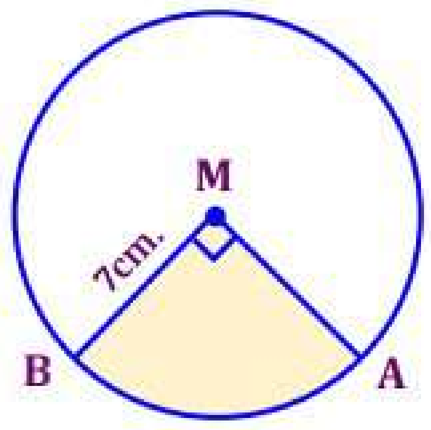


$\overline{AB}$ is a chord in circle M , $\overline{MC} \perp \overline{AB}$, D is a midpoint of $\overline{MA}$, CD = 3 cm.

, Then the surface area of the circle = ... π cm.^{2}

(a) 3 (b) 6
(c) 9 (d) 36

3 in the opposite figure : « Gharbia 2019 »

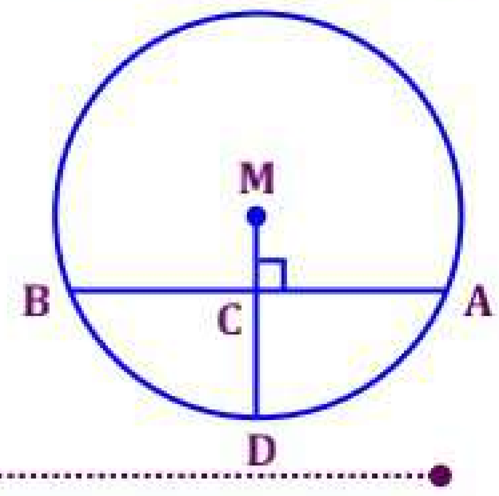


If $\overline{MA}$ and $\overline{MB}$ are two radii perpendicular in the circle M which its radius

Length = 7 cm. then the perimeter of shaded shape = ... cm. $\left(\pi = \frac{22}{7}\right)$

(a) 14 (b) 11
(c) 38.5 (d) 25

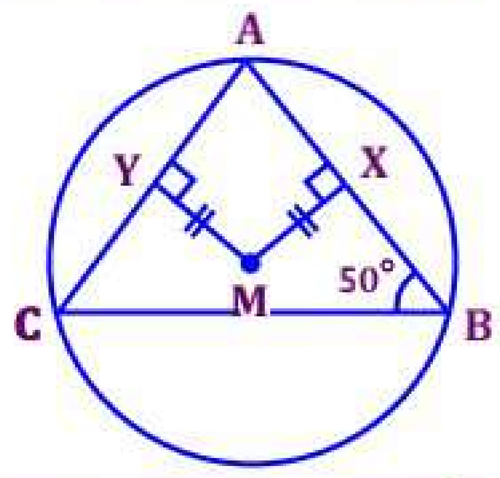
4 in the opposite figure :



Circle M with radius length = 13 cm. and AB = 24 cm. , then : CD = ... cm.

(a) 6.5 (b) 8
(c) 10 (d) 12

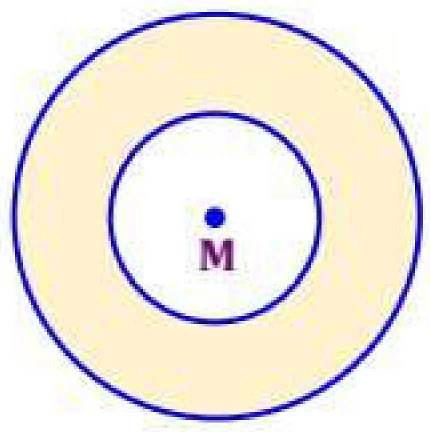
5 in the opposite figure :



MX = MY and $m(\angle B) = 50°$, then : $m(\angle A) =$ ....

(a) 50° (b) 60°
(c) 70° (d) 80°

6 in the opposite figure :

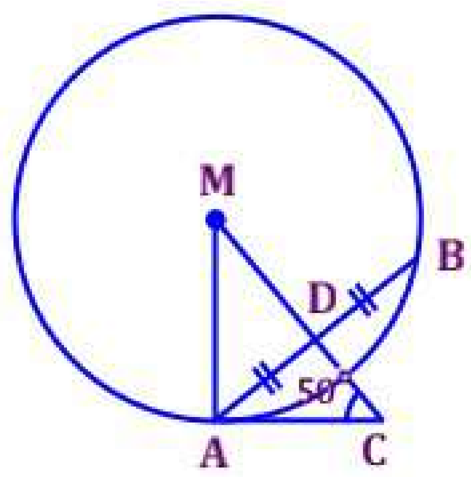


Two concentric circle with centre M , their radii 7 cm. and 14 cm.

Then the area of shaded shape = ... cm.^{2} « $\pi = \frac{22}{7}$ »

(a) 315 (b) 412
(c) 462 (d) 530

7 in the opposite figure :



AC is a tangent to the circle M at A , D is the midpoint of $\overline{AB}$ and $m(\angle C) = 50°$

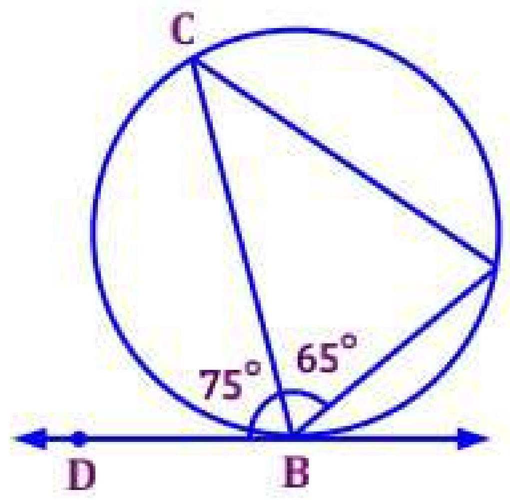
, then : $m(\angle A) =$ ....

(a) 40° (b) 45°
(c) 50° (d) 90°

8 in the opposite figure :

$\overleftrightarrow{BD}$ is a tangent to the circle M at B , $m(\angle ABC) = 65^\circ$ and $m(\angle DBC) = 75^\circ$

, Then : $m(\angle DBC) =$ ....

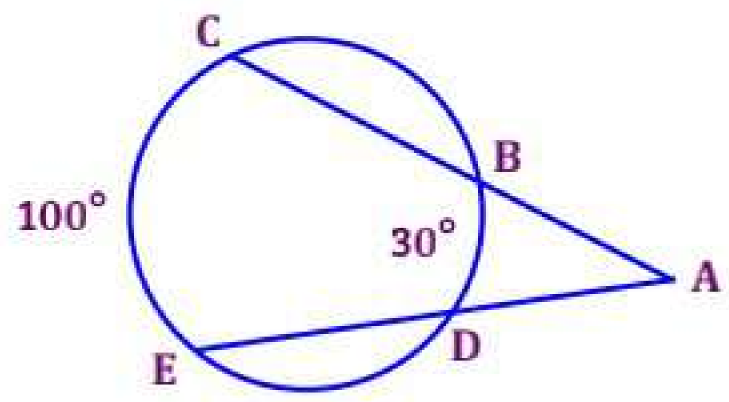


ⓐ 20° ⓑ 40°

ⓒ 50° ⓓ 80°

9 in the opposite figure :

If : $m(\overset{\frown}{CE}) = 100^\circ$ and $m(\overset{\frown}{BD}) = 30^\circ$, Then : $m(\angle A) =$ ....

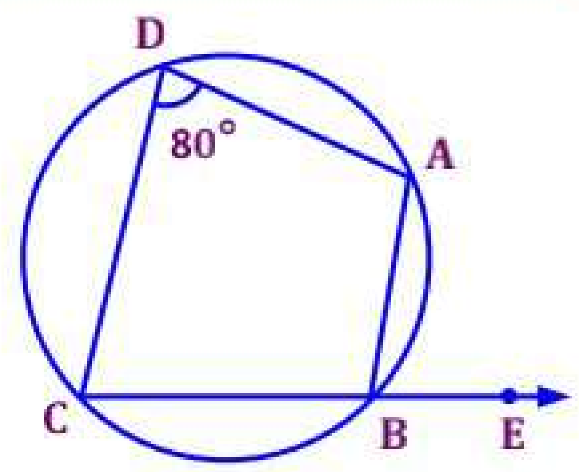


ⓐ 70° ⓑ 65°

ⓒ 60° ⓓ 35°

10 in the opposite figure :

If : $m(\angle ADC) = 80^\circ$, Then : $m(\angle ABE) =$ ....

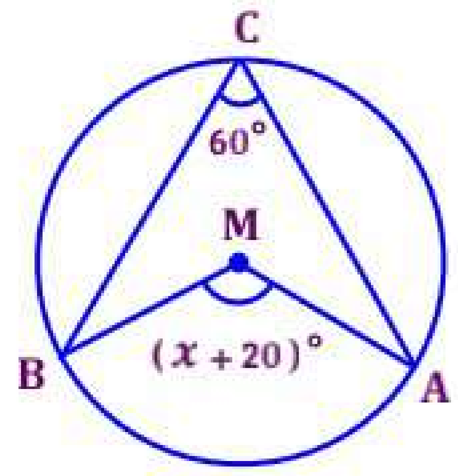


ⓐ 10° ⓑ 80°

ⓒ 60° ⓓ 100°

11 in the opposite figure :

If : $m(\angle ACB) = 60^\circ$, $(\angle AMB) = (x + 20)^\circ$, Then : $x =$ ....

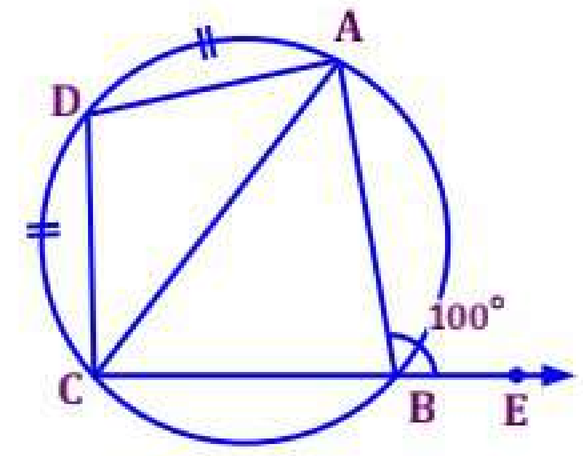


ⓐ 30° ⓑ 40°

ⓒ 80° ⓓ 100°

12 in the opposite figure :

If : $m(\angle ABE) = 100^\circ$, $m(\overset{\frown}{AD}) = m(\overset{\frown}{DC})$, Then : $m(\angle ACD) =$ ....

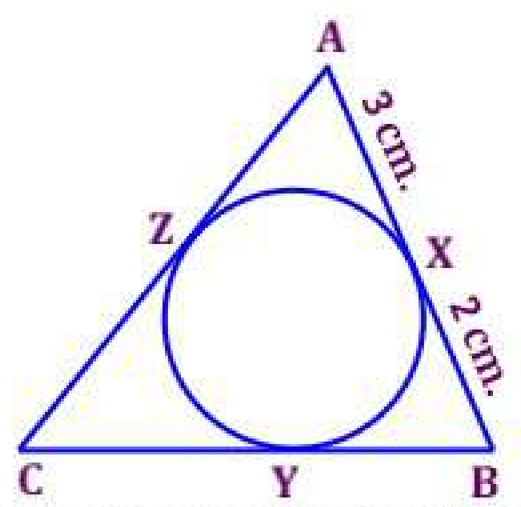


ⓐ 100° ⓑ 80°

ⓒ 40° ⓓ 30°

13 in the opposite figure :

If : AX = 3 cm. , XB = 2 cm. and , AC = 8 cm. Then : CB = ... cm.



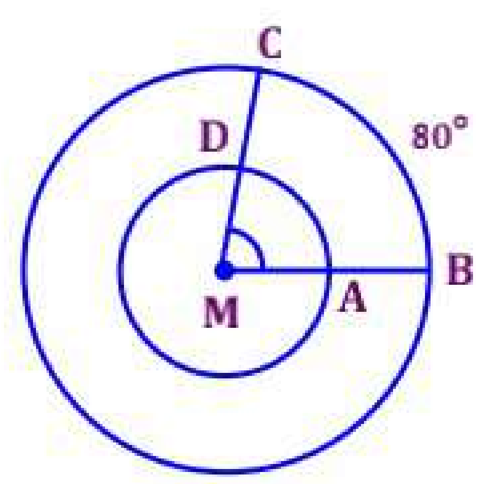
ⓐ 5 ⓑ 7

ⓒ 10 ⓓ 13

14 in the opposite figure :

Two concentric circle with centre M , $m(\overset{\frown}{AC}) = 80^\circ$

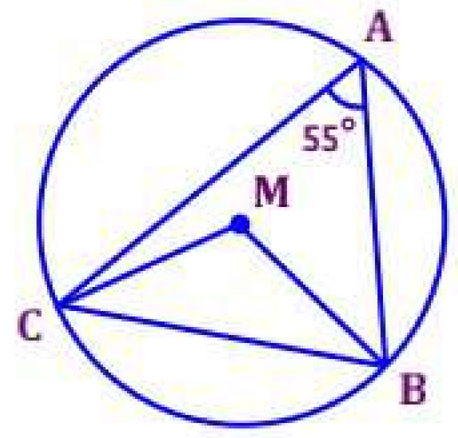
, Then : $m(\overset{\frown}{AD}) =$ ....



ⓐ 20° ⓑ 40°

ⓒ 80° ⓓ 160°

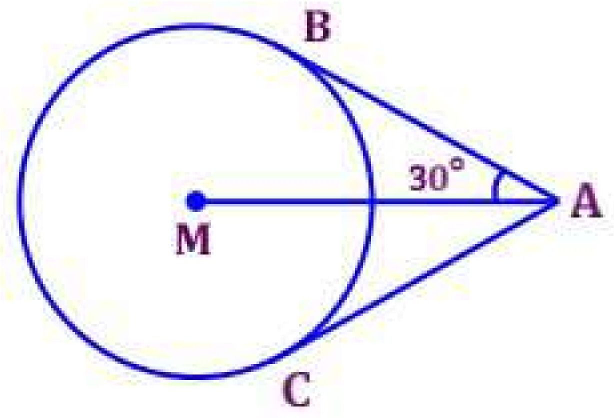
15 in the opposite figure :



If : m(∠ BAE) = 55°, Then : m(∠ MBC) = ...

ⓐ 110° ⓑ 55°
ⓒ 35° ⓓ 25°

16 in the opposite figure :

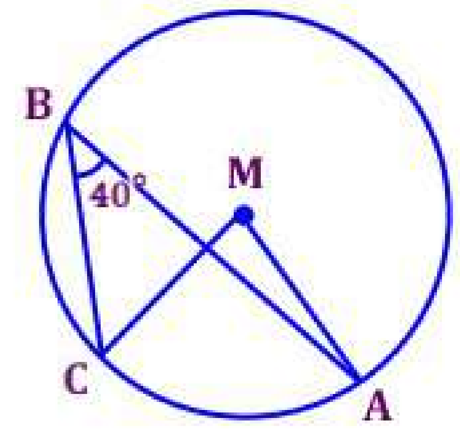


$\overline{AB}$ and $\overline{AC}$ Two tangents to the circle M from the point A , m(∠ BAM) = 30°

, Then : AB = ... cm.

ⓐ 8 ⓑ $8\sqrt{3}$
ⓒ $4\sqrt{3}$ ⓓ $2\sqrt{3}$

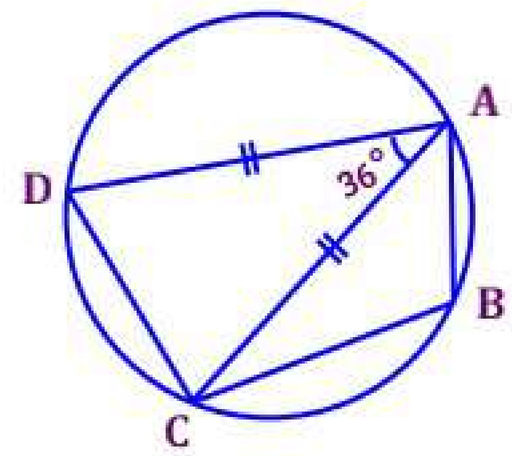
17 in the opposite figure :



If : m(∠ ABC) = 40°, Then : m(∠ AMC) = ...

ⓐ 20° ⓑ 40°
ⓒ 80° ⓓ 140°

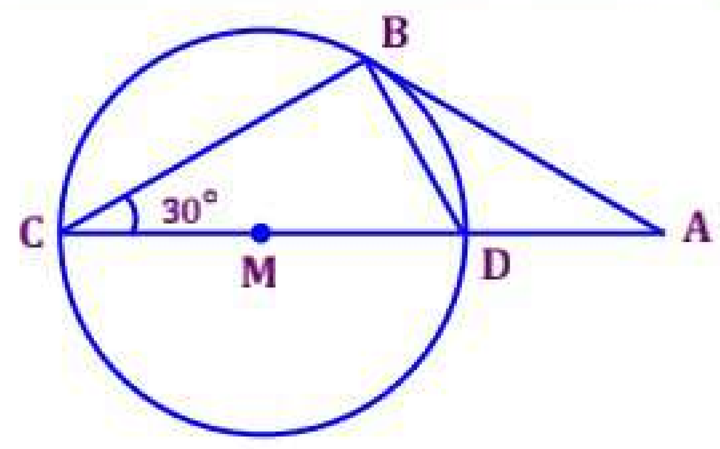
18 in the opposite figure :



If : m(∠ DAC) = 36°, and AC = AD , Then : m(∠ B) = ...

ⓐ 140° ⓑ 108°
ⓒ 70° ⓓ 40°

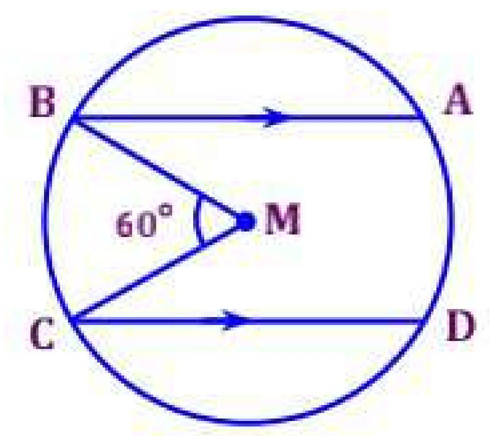
19 in the opposite figure :



$\overline{AB}$ is a diameter in circle M , m(∠ BCD) = 30°, Then : m(∠ ABC) = ...

ⓐ 120° ⓑ 110°
ⓒ 90° ⓓ 30°

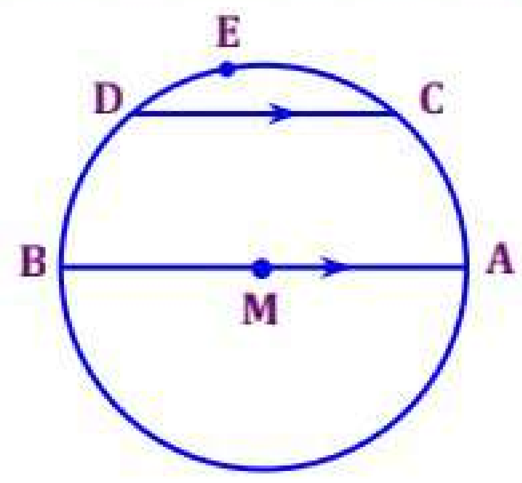
20 in the opposite figure :



$\overline{AB}$ // $\overline{CD}$, m(∠ BMC) = 60°, Then : m($\overparen{AD}$) = ...

ⓐ 30° ⓑ 60°
ⓒ 90° ⓓ 120°

21 in the opposite figure :



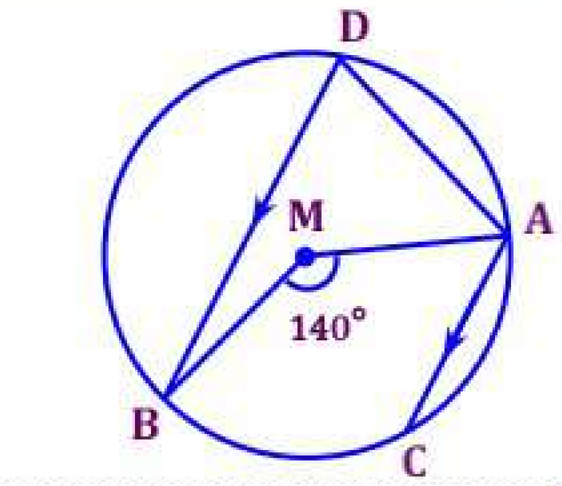
$\overline{AB}$ is a diameter in circle M , $\overline{AB}$ // $\overline{CD}$, m($\overparen{DEC}$) = 80°

, Then : m($\overparen{AC}$) = ...

ⓐ 40° ⓑ 50°
ⓒ 80° ⓓ 100°

22 in the opposite figure :

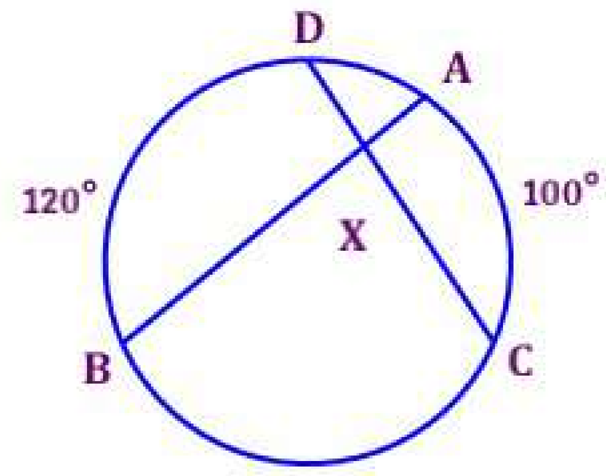
$\overline{AC} \parallel \overline{BD}$, $m(\angle AMB) = 140^\circ$, Then : $m(\angle DAC) =$ ....



ⓐ 70° ⓑ 110°
ⓒ 140° ⓓ 220°

23 in the opposite figure :

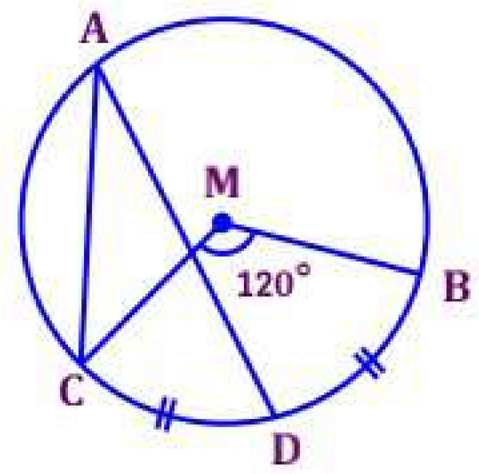
$\overline{AB} \cap \overline{DC} = \{X\}$, $m(\widehat{AC}) = 100^\circ$, $m(\widehat{BD}) = 120^\circ$, Then : $m(\angle AXC) =$ ....



ⓐ 110° ⓑ 55°
ⓒ 70° ⓓ 140°

24 in the opposite figure :

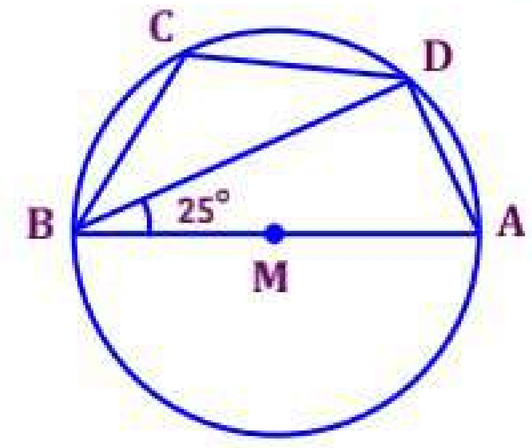
D is a midpoint of the arc $\widehat{CB}$, $m(\angle CMB) = 120^\circ$, Then : $m(\angle A) =$ ....



ⓐ 15° ⓑ 30°
ⓒ 15° ⓓ 60°

25 in the opposite figure :

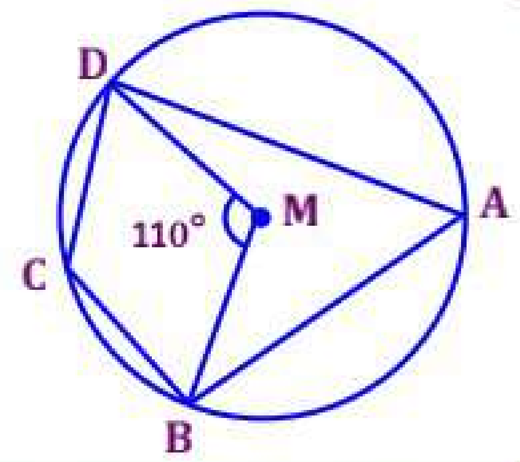
$\overline{AB}$ is a diameter in circle M , $m(\angle ABD) = 25^\circ$, Then : $m(\angle C) =$ ....



ⓐ 100° ⓑ 50°
ⓒ 115° ⓓ 125°

26 in the opposite figure :

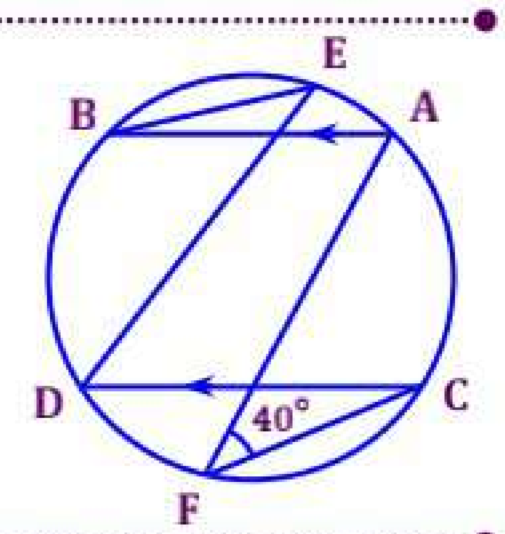
If : $m(\angle DMB) = 110^\circ$, Then : $m(\angle C) =$ ....



ⓐ 70° ⓑ 110°
ⓒ 55° ⓓ 125°

27 in the opposite figure :

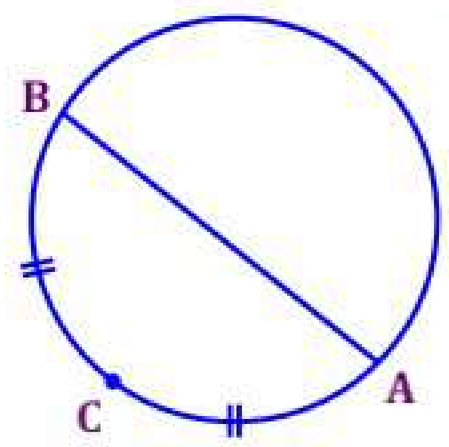
$\overline{AB} \parallel \overline{CD}$, $m(\angle AFC) = 40^\circ$, Then : $m(\angle BED) =$ ....



ⓐ 30° ⓑ 60°
ⓒ 90° ⓓ 120°

28 in the opposite figure :

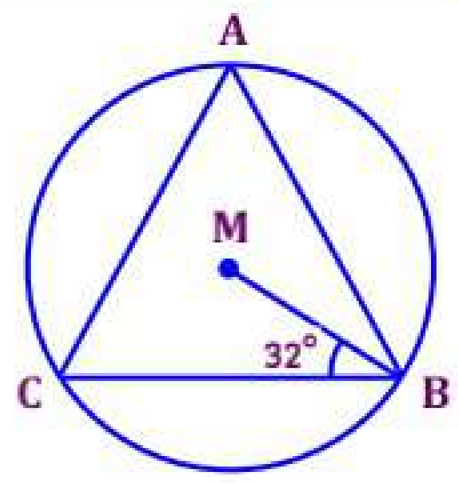
If C is a midpoint of the arc $\widehat{AB}$, Then : AB .... 2 AC



ⓐ $>$ ⓑ $<$
ⓒ $=$ ⓓ $\leqslant$

29 in the opposite figure :

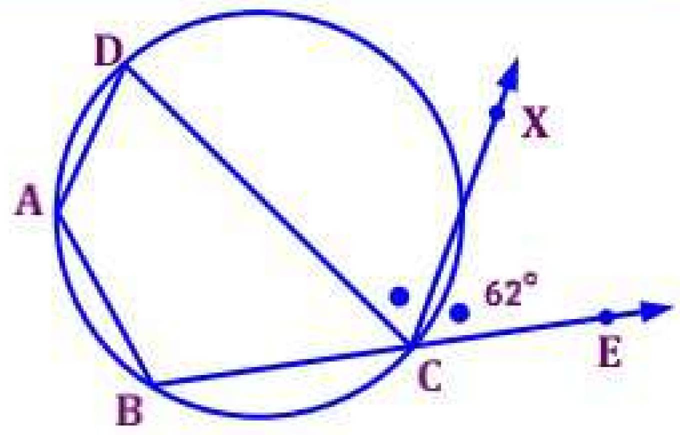
If : $m(\angle MBC) = 32^\circ$, Then : $m(\angle A) =$ ....



ⓐ 16° ⓑ 32°
ⓒ 116° ⓓ 64°

30 in the opposite figure :

$\overrightarrow{CX}$ bisects $\angle$ DCE and m ($\angle$ ECX) = 62°, Then : m ($\angle$ A) = ...

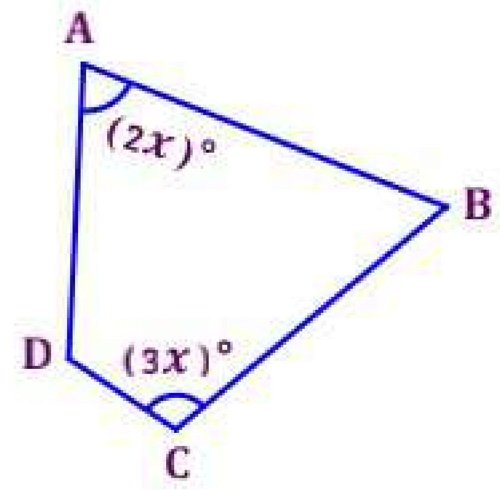


ⓐ 62° ⓑ 128°
ⓒ 56° ⓓ 124°

31 in the opposite figure :

ABCD is a cyclic quadrilateral , in which : m ($\angle$ A) = $(2x)°$, ($\angle$ C) = $(3x)°$

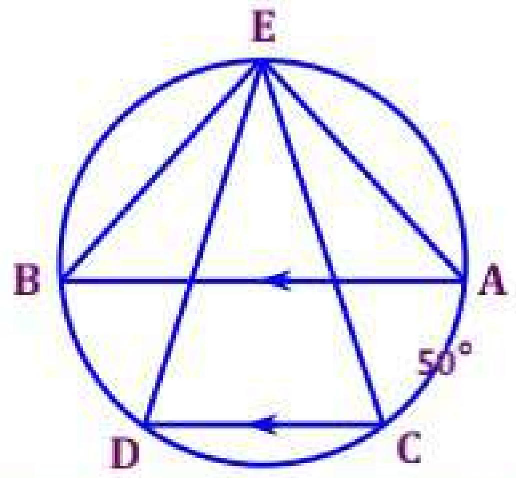
, then : x = ... « R.sea 2016 »



ⓐ 20° ⓑ 32°
ⓒ 32° ⓓ 36°

32 in the opposite figure :

$\overline{AB}$ // $\overline{CD}$, m ($\overset{\frown}{AC}$) = 50°, Then : m ($\angle$ BED) = ...

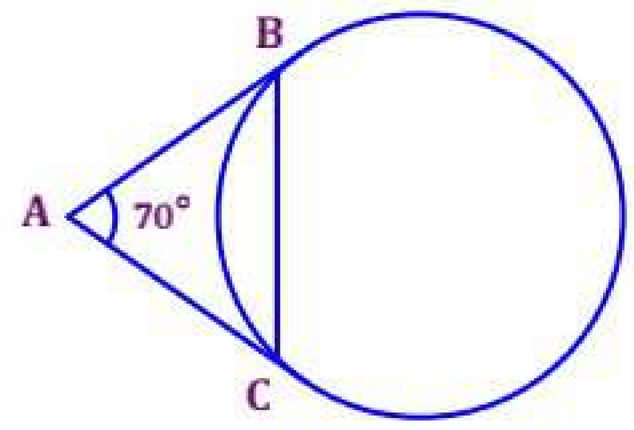


ⓐ 50° ⓑ 5°
ⓒ 25° ⓓ 20°

33 in the opposite figure :

$\overline{AB}$ and $\overline{AC}$ Two tangents to the circle M from the point A , m ($\angle$ BAM) = 70°

, Then : m ($\overset{\frown}{AC}$) = ...

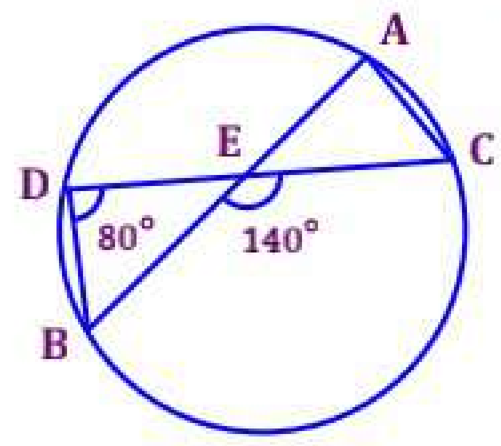


ⓐ 150° ⓑ 110°
ⓒ 100° ⓓ 90°

34 in the opposite figure :

$\overline{AB} \cap \overline{DC}$ = { E }, m ($\angle$ CEM) = 140°, m ($\angle$ CDB) = 80°

, Then : m ($\angle$ ACD) = ...

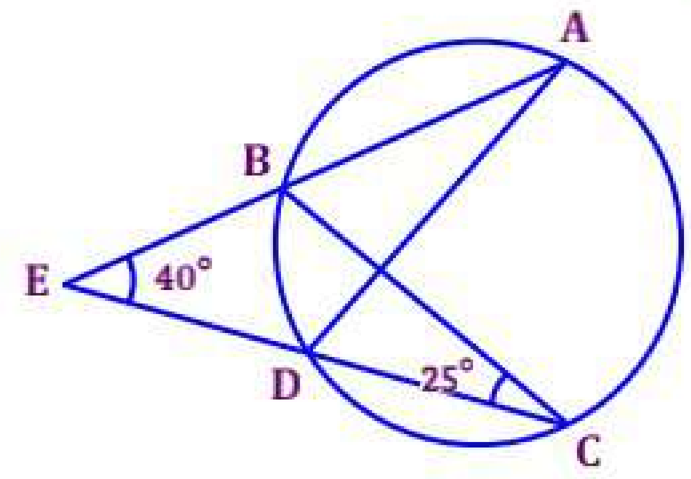


ⓐ 30° ⓑ 40°
ⓒ 50° ⓓ 60°

35 in the opposite figure :

$\overrightarrow{AB} \cap \overrightarrow{DC}$ = { E }, m ($\angle$ E) = 40°, m ($\angle$ DCB) = 25°

, Then : m ($\angle$ ABC) = ...

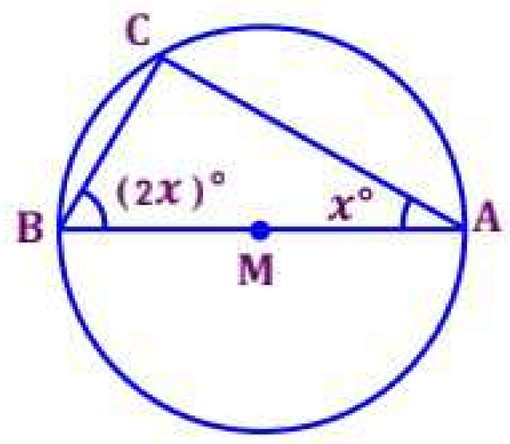


ⓐ 50° ⓑ 80°
ⓒ 25° ⓓ 65°

36 in the opposite figure :

$\overline{AB}$ is a diameter in circle M , m ($\angle$ CAB) = $x°$, m ($\angle$ CBA) = $x°$

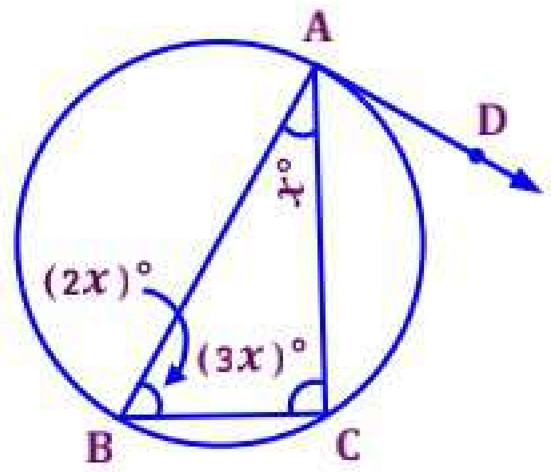
, Then : x = ...



ⓐ 20° ⓑ 30°
ⓒ 40° ⓓ 60°

37 in the opposite figure :

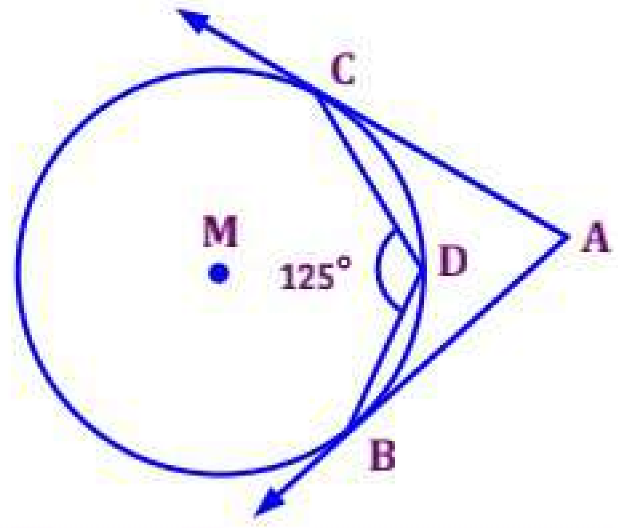
$\overline{AD}$ touches the circle M at A , m(∠ CAB) = $x°$, m(∠ CBA) = $2x°$
, m(∠ ACB) = $3x°$, Then : m(∠ DAC) = ...



ⓐ 20° ⓑ 40°
ⓒ 60° ⓓ 80°

38 in the opposite figure :

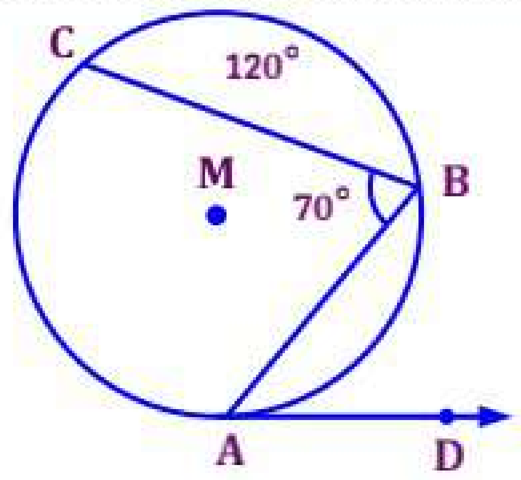
$\overline{AB}$ and $\overline{AC}$ Two tangents to the circle M from the point A , m(∠ CDB) = 125°
, Then : m(∠ A) = ...



ⓐ 50° ⓑ 60°
ⓒ 70° ⓓ 80°

39 in the opposite figure :

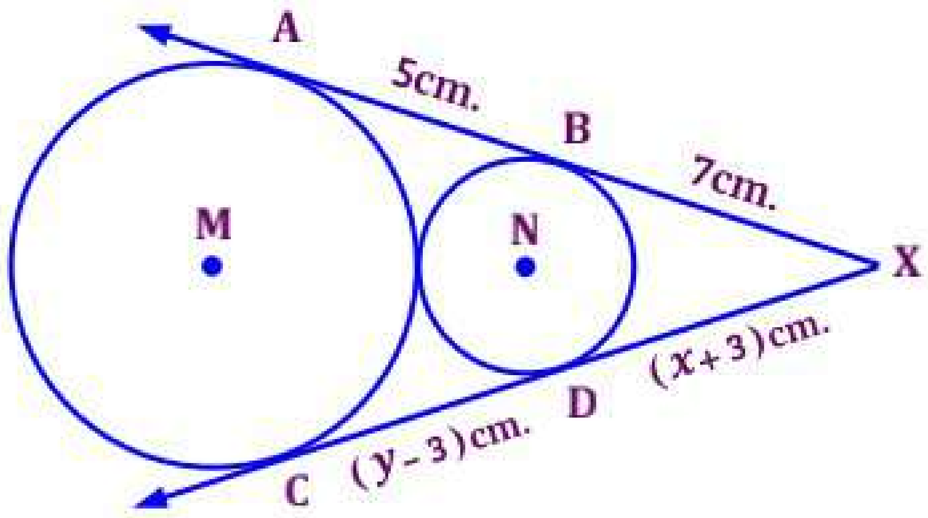
$\overline{AD}$ touches the circle M at A , m(∠ CAB) = 70° , m($\overset{\frown}{BC}$) = 120°
, Then : m(∠ A) = ...



ⓐ 50° ⓑ 60°
ⓒ 35° ⓓ 70°

40 in the opposite figure :

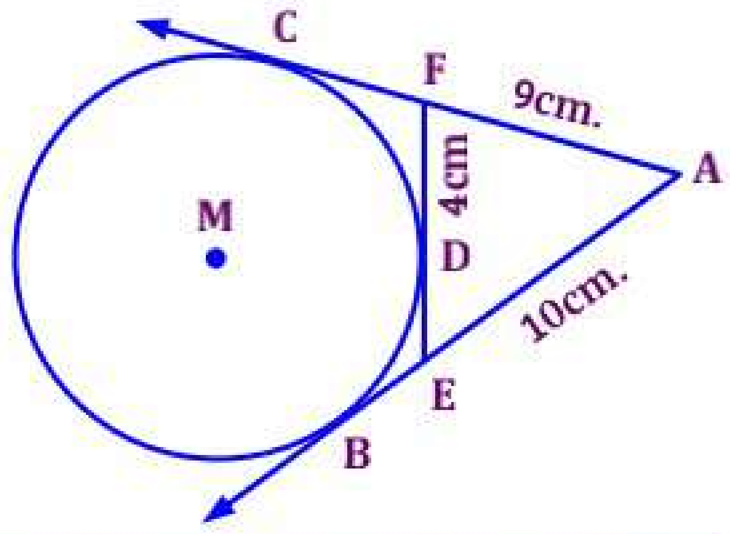
$\overline{AB}$ and $\overline{AC}$ Two to the two circles M and N from the point X
BX = 7 cm. , DX = $(x + 3)$ cm. , AB = 5 cm. and CD = $(y - 3)$ cm.
, Then : $x + y$ = ...



ⓐ 10 ⓑ 11
ⓒ 12 ⓓ 14

41 in the opposite figure :

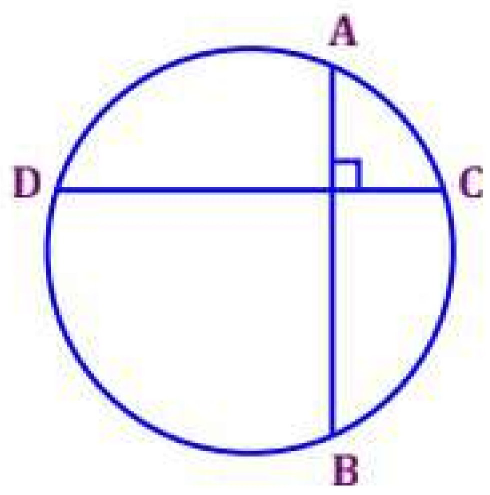
$\overline{AB}$ and $\overline{AC}$ Two tangents to the circle M from the point A
FD = 4 cm. , FA = 9 cm. , AE = 10 cm. , Then : ED = ... cm.



ⓐ 3 ⓑ 4
ⓒ 5 ⓓ 6

42 in the opposite figure :

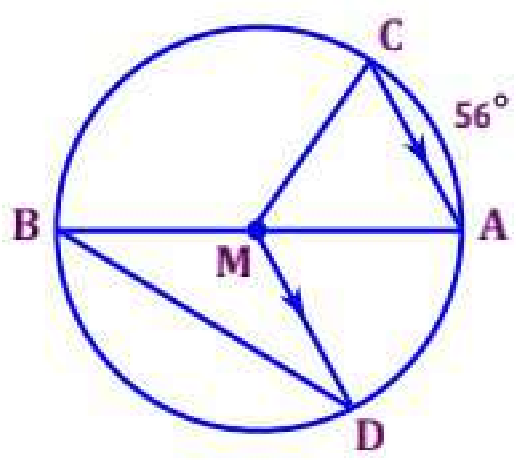
$\overline{AB}$ and $\overline{DC}$ are two radii perpendicular in the circle M ,
, Then : m($\overset{\frown}{AC}$) + m($\overset{\frown}{BD}$) = ...



ⓐ 45° ⓑ 90°
ⓒ 180° ⓓ 270°

43 in the opposite figure :

$\overline{AC}$ // $\overline{DM}$ and m($\overset{\frown}{AC}$) = 56° , then : m(∠ ABD) = ...

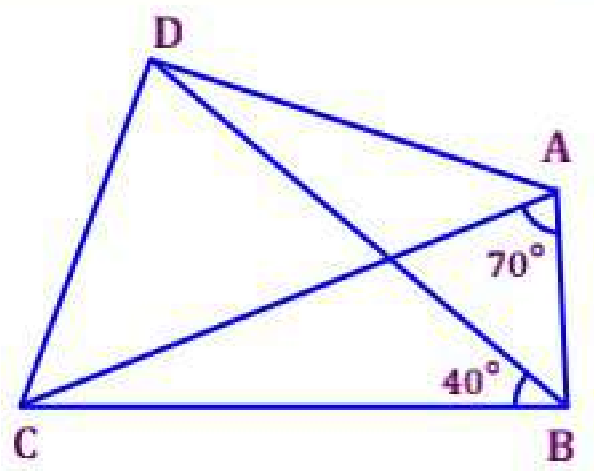


ⓐ 28° ⓑ 56°
ⓒ 62° ⓓ 31°

44 in the opposite figure :

ABCD is a cyclic quad , in which : $m(\angle BAC) = 70^\circ$, $(\angle DBC) = 40^\circ$

, Then : $m(\angle DCB) =$ ....

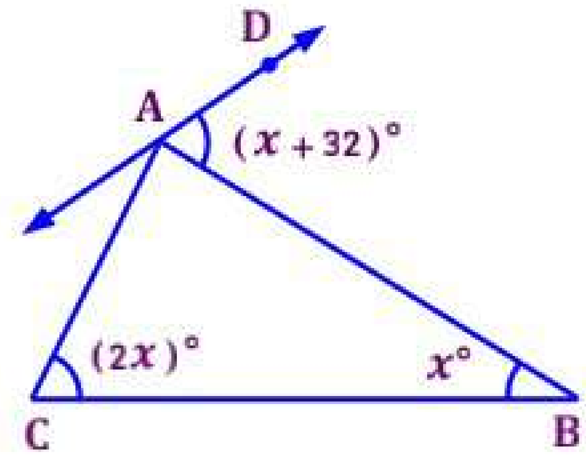


ⓐ 30° ⓑ 40°
ⓒ 70° ⓓ 110°

45 in the opposite figure :

If : $\overline{AD}$ touches the circle whose passes through the vertices of the triangle ABC at A , $m(\angle DAB) = (x + 32)^\circ$, $m(\angle ACB) = (2x)^\circ$, $m(\angle ABC) = x^\circ$

, Then : $m(\angle BAC) =$ ....

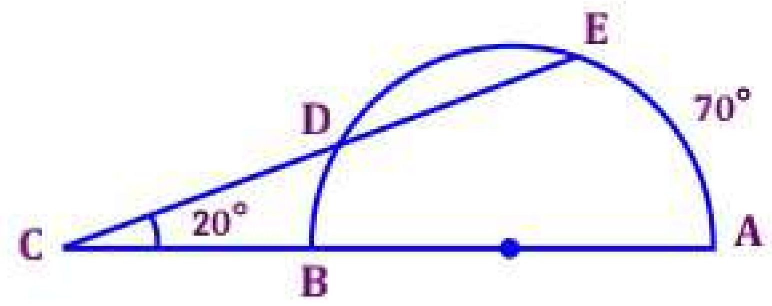


ⓐ 32° ⓑ 64°
ⓒ 84° ⓓ 42°

46 in the opposite figure :

$\overline{AB}$ is a diameter in circle M , $m(\angle ECA) = 20^\circ$, $m(\widehat{AE}) = 70^\circ$

, Then : $m(\widehat{DE}) =$ ....

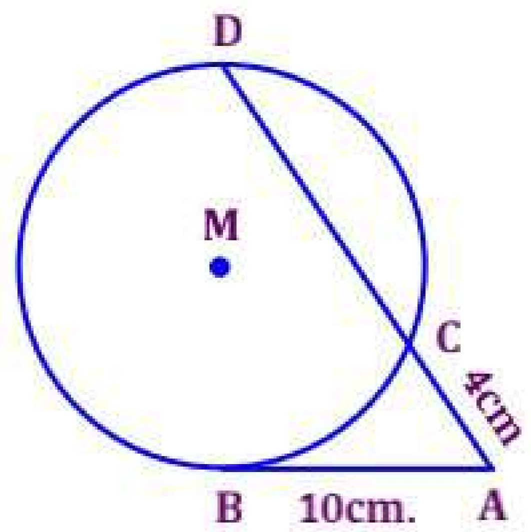


ⓐ 70° ⓑ 80°
ⓒ 90° ⓓ 110°

47 in the opposite figure :

$\overline{AB}$ is a tangent to the circle at B , AB = 10 cm. , AC = 4 cm.

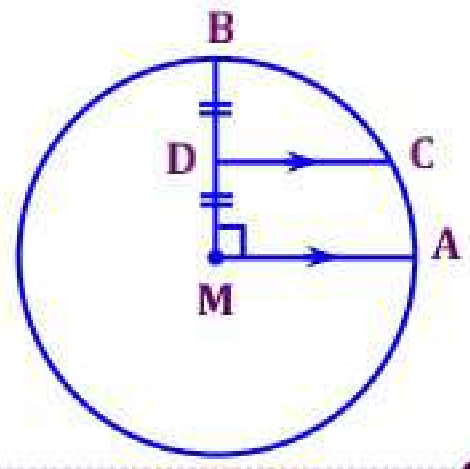
, Then : DC = ... cm.



ⓐ 4 ⓑ 6
ⓒ 9 ⓓ 10

48 in the opposite figure :

$\overline{DC} \parallel \overline{AM}$ and D is a midpoint of $\overline{BM}$, Then : $m(\widehat{AC}) =$ ....

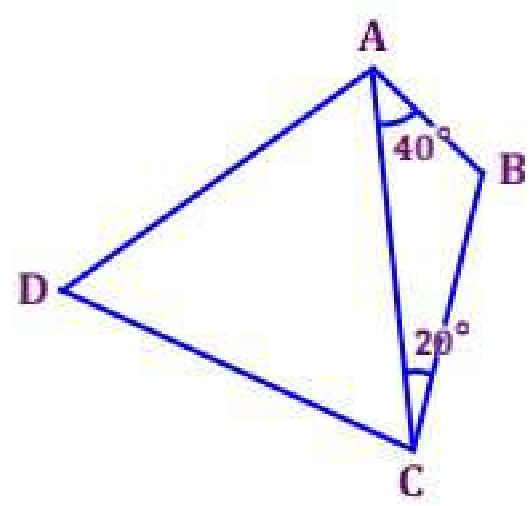


ⓐ 60° ⓑ 30°
ⓒ 45° ⓓ 90°

49 in the opposite figure :

$\overline{ABCD}$ is a cyclic quad , in which : $m(\angle BAC) = 40^\circ$, $(\angle ACB) = 20^\circ$

, Then : $m(\angle D) =$ ....



ⓐ 20° ⓑ 40°
ⓒ 60° ⓓ 120°

Geometry for prep (3)

① Prove that

$\cos 60 = \cos^2 30 - \sin^2 30$

sol

$l.H.S = \cos 60 = \frac{1}{2}$

$R.H.S = \cos^2 30 - \sin^2 30$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{1}{2}$

$l.H.S = R.H.S$

② Prove that

$\tan 60 = 2\tan 30 \div (1 - \tan^2 30)$

$l.H.S = \tan 60 = \sqrt{3}$

$R.H.S =$

$2\tan 30 \div (1 - \tan^2 30)$

$= 2 \times \frac{1}{\sqrt{3}} \div \left(1 - \left(\frac{1}{\sqrt{3}}\right)^2\right)$

$= \sqrt{3}$

$l.H.S = R.H.S$

③ Prove that

$\sin^3 30 = 9\cos^3 60 - \tan^2 45$

sol

$l.H.S = \sin^3 30 = \left(\frac{1}{2}\right)^3$

$= \frac{1}{8}$

$R.H.S = 9\cos^3 60 - \tan^2 45$

$= 9 \times \left(\frac{1}{2}\right)^3 - (1)^2$

$= \frac{1}{8}$

$l.H.S = R.H.S$

④ Prove that

$\sin 60 = 2\sin 30 \cos 30$

$l.H.S = \sin 60 = \frac{\sqrt{3}}{2}$

$R.H.S = 2\sin 30 \cos 30$

$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

$\therefore l.H.S = R.H.S$

⑤ Find the value of

$\cos 60 * \sin 30 - \sin 60 \cos 30$

$= \frac{1}{2} * \frac{1}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} - \frac{3}{4} = \frac{-1}{2}$

Mr. HASSAN SALAMA 01141060113 / 01283914718

6) find the value of

$$\frac{\cos^2 60 + \cos^2 30 + \tan^2 45}{\sin 60 \tan 60 - \sin 30}$$

$$= \frac{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 + (1)^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - \frac{1}{2}}$$

$$= 2$$

7) find the value of X if $0 < X < 90$

$$\sin X \ \sin 45 \cos 45 \ \tan 60 = \tan^2 45 - \cos^2 60$$

Sol

$$\sin x \cdot \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sqrt{3} = (1)^2 - (\frac{1}{2})^2$$

$$\frac{\sqrt{3}}{2} \sin x = \frac{3}{4} \quad \div \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\therefore x = 60$$

8) $\sin 2x = \sin 60 \cos 30 - \cos 60 \sin 30$

Sol

$$\sin 2x = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$\sin 2x = \frac{1}{2}$$

$$\therefore 2x = 30 \quad \div 2$$

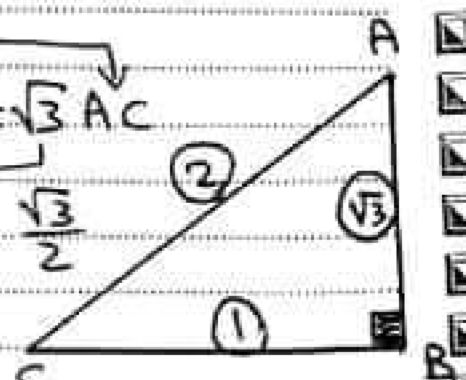
$$x = 15$$

9) ABC is a right angled Triangle at B $2AB = \sqrt{3}\,AC$ find the trigonometrical ratios for angle C

Sol

$$\because 2AB = \sqrt{3}\,AC$$

$$\therefore \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$



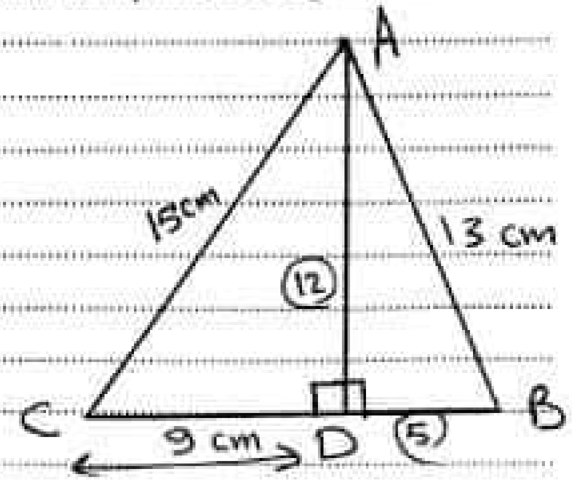
$$BC = \sqrt{(2)^2 - (\sqrt{3})^2} = 1$$

$$\sin C = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{1}{2}$$

$$\tan C = \frac{\sqrt{3}}{1} = \sqrt{3}$$

10 In the opposite figure



Find the value of

$$\frac{\tan(C\hat{A}D) + \tan(B\hat{A}D)}{\tan(C\hat{A}D) - \tan(B\hat{A}D)}$$

Sol In $\triangle ACD$

$AD = \sqrt{(15)^2 - (9)^2} = 12\text{cm}$

in $\triangle ADB$

$BD = \sqrt{(13)^2 - (12)^2} = 5\text{cm}$

* the expression

$$= \frac{\frac{9}{12} + \frac{5}{12}}{\frac{9}{12} - \frac{5}{12}}$$

$$= \boxed{\frac{7}{2}}$$

11 ABCD is a Trapezuid in which $\overline{AD} \parallel \overline{BC}$, $m(\hat{B}) = 90^\circ$

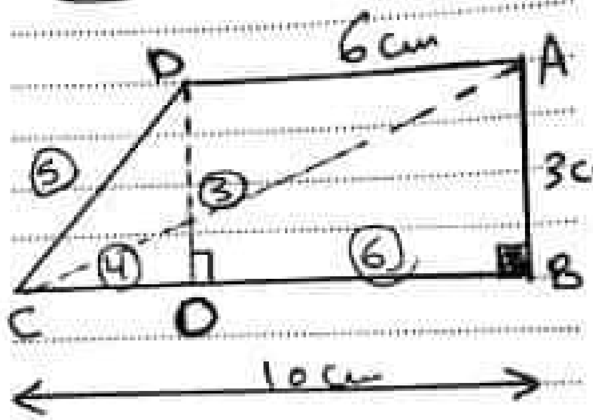
$AB = 3\text{cm}$, $AD = 6\text{cm}$

$BC = 10\text{ cm}$

prove that

$\cos(D\hat{C}B) - \tan(A\hat{C}B) = \frac{1}{2}$

Sol



Draw $\overline{DO} \perp \overline{BC}$

ABOD is rectangle

$\therefore OB = AD = 6\text{cm}$

$DO = AB = 3\text{cm}$

$CO = 10 - 6 = 4\text{cm}$

$DC = \sqrt{3^2 + 4^2} = 5\text{cm}$

$\cos(D\hat{C}B) - \tan(A\hat{C}B)$

$= \frac{4}{5} - \frac{3}{10}$

$= \frac{1}{2} = \text{R.H.S}$

[12] ABC is a triangle where AB = AC = 10 cm, BC = 12 cm

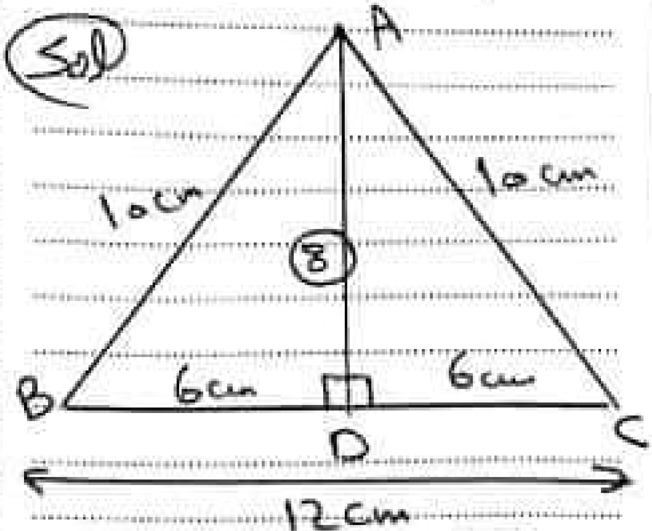
Find

(1) $m(\angle B)$

(2) prove that $\sin B + \cos C = 1.4$

(3) prove that $\sin^2 C + \cos^2 C = 1$

Sol



Draw $\overline{AD} \perp \overline{BC}$

(1) $\cos B = \frac{6}{10} = \frac{3}{5}$

$\therefore m(\angle B) = 53^\circ 7' 49''$

(2) $\sin B + \cos C = \frac{8}{10} + \frac{6}{10}$

$= \frac{14}{10} = 1.4$

(3) $\sin^2 C + \cos^2 C$

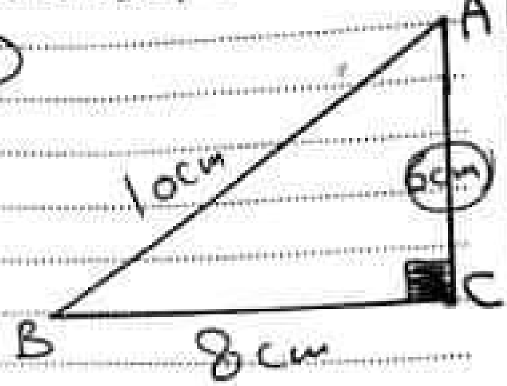
$= \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2 = 1$

[13] ABC is right angled triangle at C where AB = 10 cm, BC = 8 cm

prove that

$\sin A \cos B + \cos A \sin B = 1$

Sol



$AC = \sqrt{(10)^2 - (8)^2} = 6$ cm

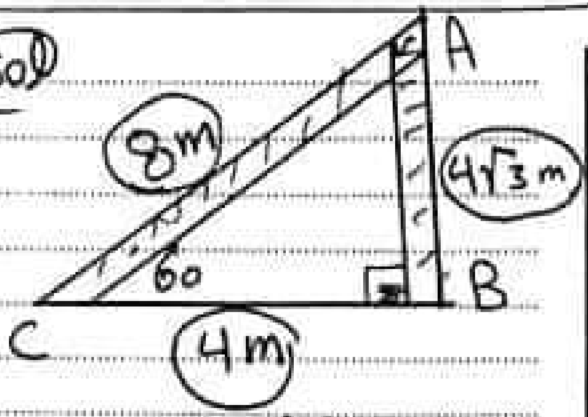
$\sin A \cos B + \cos A \sin B$

$= \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10} = 1$

$=$ R.H.S

[14] due to the wind the upper part of a tree was broken and make with the horizontal an angle of measure 60° if the distance between the top of the tree and the base is 4 m find the length of the tree

Sol



$\frac{\tan 60}{1} = \frac{AB}{4}$

$\therefore AB = 4 \tan 60 = 4\sqrt{3}$ m

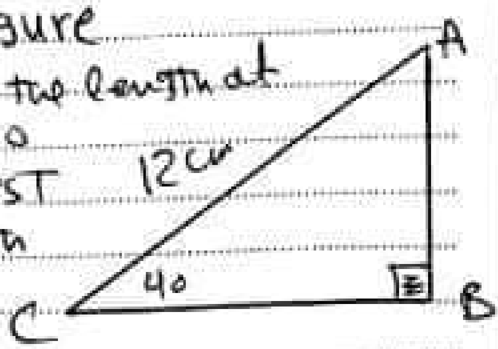
$\therefore AC = \sqrt{(4)^2 + (4\sqrt{3})^2} = 8$ m

the length of the tree

$= 4\sqrt{3} + 8 \simeq 15$ m

15 in the opposite figure

Find the length of $\overline{AB}$ to nearest tenth

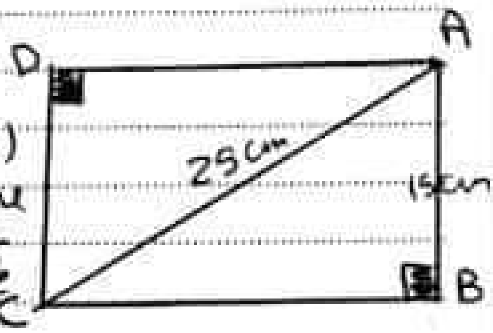


Sol $\frac{\sin 40}{1} = \frac{AB}{12}$

$\therefore AB = 12 \sin 40 \simeq 7.7$ cm

16 in the opposite figure

Find $m(A\hat{C}B)$ and the area of rectangle



Sol $\sin(A\hat{C}B) = \frac{15}{25}$

$= \frac{3}{5}$

$\therefore m(A\hat{C}B) = 36°52'11''$

$BC = \sqrt{(25)^2 - (15)^2} = 20$ cm

area of rectangle

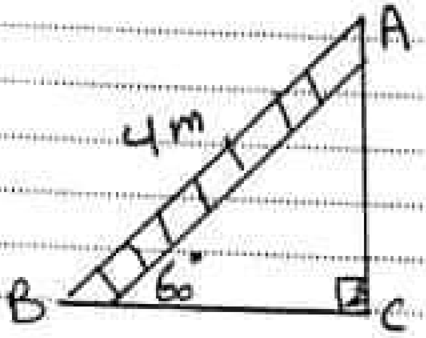
$= L \times W = 20 \times 15$

$= 300 \text{ cm}^2$

17 $\overline{AB}$ is a ladder of length 4 meters its upper end A stand at a vertical wall and its lower end B on an horizontal ground and the measure of angle of inclination of the ladder on the ground is 60°

Find $\overline{AC}$ length

where AC is the distance between upper end and ground



$\frac{\sin 60}{1} = \frac{AC}{4}$

$AC = 4\sin 60 = 2\sqrt{3}$ m

(18) if

$\tan x = 4\cos 30 - \tan 60$

find x

sol $\tan x = 4 \times \frac{\sqrt{3}}{2} - \sqrt{3}$

$\tan x = \sqrt{3}$

$x = 60$

(19) $2\sin A = \tan^2 60 - 2\tan 45$

sol $2\sin A = (\sqrt{3})^2 - 2 \times 1$

$2\sin A = 1$

$\therefore \sin A = \frac{1}{2}$

$A = 30$

(20) if $\sin \frac{x}{3} = \frac{1}{2}$

then $x = ...$

$\therefore \sin \frac{x}{3} = \frac{1}{2}$

$\therefore \frac{x}{3} = 30$

$\therefore x = 3 \cdot 30 = 90^\circ$

(21) if the ratio between the measures of the interior angles of a triangle is 3:4:7 find the degree measure of each angle

sol let the measures of angles

$3x$, $4x$ and $7x$

$\therefore 3x + 4x + 7x = 180$

$14x = 180$ $(\div 14)$

$x = \frac{90}{7}$

the measure of

(1) first angle $= 3 \times \frac{90}{7} =$

(2) 2nd angle $= 4 \times \frac{90}{7}$

(3) 3rd angle $= 7 \times \frac{90}{7}$

$= 90^\circ$

22 Two supplementary angles the ratio between their measure 3 : 5 find the degree measure of each angle

Sol

let the measures of angles $3X$ and $5X$

$3X + 5X = 180$

$8X = 180$ (÷8)

$X = 22.5$

The measure of the first angle $= 3 \times 22.5 = 67°30'$

the measure of the second angle $= 5 \times 22.5$

$= 112°30'$

23 if

$X \sin 45 \cos 45 \tan 60 = \tan^2 45 - \cos^2 60$

find X

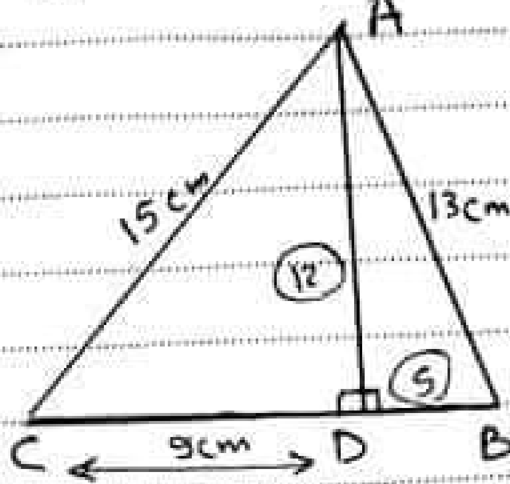
Sol

$X \cdot \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sqrt{3} = 1^2 - (\frac{1}{2})^2$

$\frac{\sqrt{3}}{2} x = \frac{3}{4}$ (÷ $\frac{\sqrt{3}}{2}$)

$x = \frac{\sqrt{3}}{2}$

24 in the opposite figure



find the value of

$$\frac{\tan(\hat{CAD}) + \tan(\hat{BAD})}{\tan(\hat{CAD}) - \tan(\hat{BAD})}$$

Sol

in ΔACD

$AD = \sqrt{15^2 - 9^2}$

$= 12$ cm

in ΔADB

$BD = \sqrt{13^2 - 12^2} = 5$ cm

$$\frac{\tan(\hat{CAD}) + \tan(\hat{BAD})}{\tan(\hat{CAD}) - \tan(\hat{BAD})}$$

$$= \frac{\frac{9}{12} + \frac{5}{12}}{\frac{9}{12} - \frac{5}{12}}$$

$$= \frac{7}{2}$$

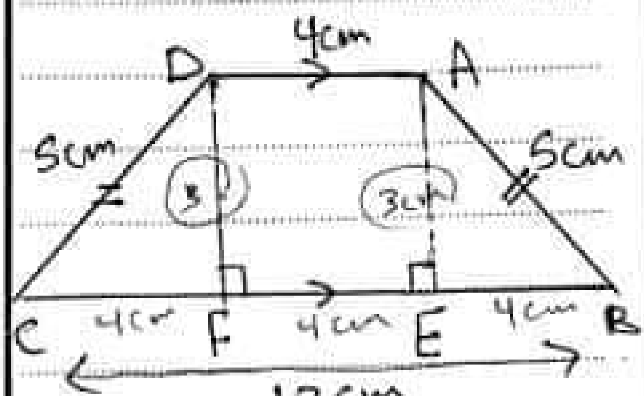
Mr. HASSAN SALAMA 01141060113 / 01283914718

(25) ABCD is an isosceles Trapezium $\overline{AD} // \overline{BC}$, AD = 4 cm, AB = 5 cm, BC = 12 cm

Prove that

$$\frac{5\tan B \cos C}{\sin^2 C + \cos^2 B} = 3$$

Sol



the figure AEFD is a rectangle

$\therefore$ FE = DA = 4 cm

$\therefore BE = CF = \frac{12-4}{2} = 4$ cm

$AE = \sqrt{5^2 - 4^2} = 3$ cm

$$\frac{5\tan B \cos C}{\sin^2 C + \cos^2 B} = \frac{5 \times \frac{3}{4} \times \frac{4}{5}}{(\frac{3}{5})^2 + (\frac{4}{5})^2} = \boxed{3}$$

(26) if

$2\sin A = \tan^2 60 - 2\tan 45$

Sol

$2\sin A = (\sqrt{3})^2 - 2 \times 1$

$2\sin A = 1 \quad (\div 2)$

$\sin A = \frac{1}{2}$

$m(\hat{A}) = 30^\circ$

(27) if

$x^2 = \cos 60 \sin 30 + \cos 30 \sin 60$

Sol

$x^2 = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = 1$

$x = \pm 1$

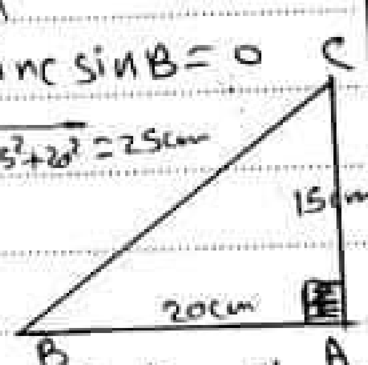
(28) ABC is a right angled triangle at A, AC = 15 cm, AB = 20 cm

find ① 2 sin C cos C ② tan C × tan B

③ Prove that

cos C cos B − sin C sin B = 0

Sol $BC = \sqrt{15^2 + 20^2} = 25$ cm



① $2\sin C \cos C = 2 \times \frac{20}{25} \times \frac{15}{25} = \frac{24}{25}$

② $\tan C \times \tan B = \frac{20}{15} \times \frac{15}{20} = 1$

③ $\cos C \cos B - \sin C \sin B$

$= \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25} = 0$

(29) Prove that ΔABC is right angled at B then find its area where $A(1,4)$, $B(-1,-2)$, $C(2,-3)$

Sol

$AB = \sqrt{(1+1)^2+(4+2)^2} = \sqrt{40}$ l.u

$BC = \sqrt{(-1-2)^2+(-2+3)^2} = \sqrt{10}$ l.u

$AC = \sqrt{(1-2)^2+(4+3)^2} = \sqrt{50}$ l.u

$(AC)^2 = (\sqrt{50})^2 = 50$

$(AB)^2+(BC)^2 = (\sqrt{40})^2+(\sqrt{10})^2 = 50 = (AC)^2$

$\therefore$ ABC is right angled at B

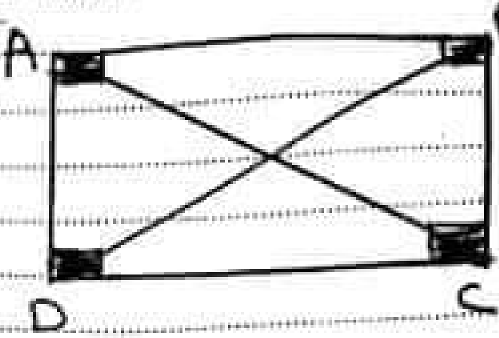
area of $\Delta = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times \sqrt{40} \times \sqrt{10}$

$= 10$ squared unit

(30) prove that the points $A(1,0)$, $B(-1,4)$, $C(7,8)$, $D(9,4)$ are a vertices of rectangle and find its diagonal length.

Sol



$AB = \sqrt{(1+1)^2+(0-4)^2} = \sqrt{20}$ l.u

$BC = \sqrt{(-1-7)^2+(4-8)^2} = 4\sqrt{5}$ l.u

$CD = \sqrt{(7-9)^2+(8-4)^2} = \sqrt{20}$ l.u

$DA = \sqrt{(9-1)^2+(4-0)^2} = 4\sqrt{5}$ l.u

$AC = \sqrt{(7-1)^2+(8-0)^2} = 10$ l.u

$BD = \sqrt{(9+1)^2+(4-4)^2} = 10$ l.u

$\therefore AB = CD$

$BC = DA$

$AC = BD$

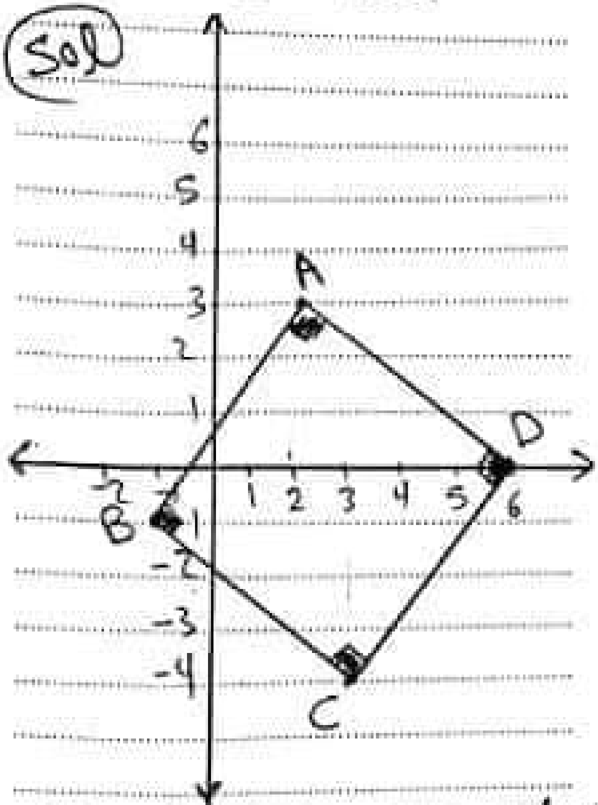
$\therefore$ ABCD is a rectangle

its diagonal length

$AC = BD = 10$ length unit

(31) represent graphically on the diagram coordinates the points A(2,3), B(-1,-1) C(3,-4), D(6,0) then prove that ABCD is a square and find its area

(Sol)



$AB = \sqrt{(2+1)^2+(3+1)^2} = 5$ l.u

$BC = \sqrt{(-1-3)^2+(-1+4)} = 5$ l.u

$CD = \sqrt{(3-6)^2+(-4-0)^2} = 5$ l.u

$DA = \sqrt{(2-6)^2+(3-0)^2} = 5$ l.u

$AC = \sqrt{(2-3)^2+(3+4)^2} = \sqrt{50}$ l.u

$BD = \sqrt{(-1-6)^2+(-1-0)^2} = \sqrt{50}$ l.u

$\because AB = BC = CD = DA$

$AC = BD$

$\therefore$ ABCD is a square

its area $= S \times S$

$= 5 \times 5 = 25$ squared unit

(32) Prove that $\triangle ABC$ where A(1,-2), B(-4,2) C(1,6) is an isosceles triangle

(Sol)

$AB = \sqrt{(1+4)^2+(-2-2)^2} = \sqrt{41}$ l.u

$BC = \sqrt{(-4-1)^2+(2-6)^2} = \sqrt{41}$ l.u

$AC = \sqrt{(1-1)^2+(-2-6)^2} = 8$ l.u

$\because AB = BC$

$\therefore \triangle ABC$ is an isosceles

(33) if the distance between (x,5) and (6,1) is $2\sqrt{5}$ length unit find x

(Sol)

$\sqrt{(x-6)^2+(5-1)^2} = 2\sqrt{5}$

$\sqrt{(x-6)^2+16} = 2\sqrt{5}$ by squaring

$(x-6)^2 + 16 = 20$

$(x-6)^2 = 4$

$\therefore x-6 = \pm 2$

$x-6=2$	$x-6=-2$
$x=2+6$	$x=-2+6$
$=8$	$=4$

(34) Prove that the points A(3, −1), B(−4, 6), C(2, −2) lie on a circle of centre M(−1, 2) and find its circumference $\pi = 3.14$

Sol

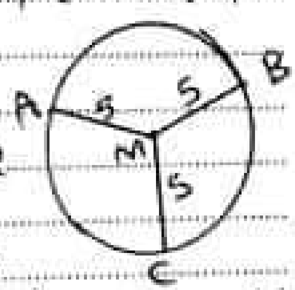
$MA = \sqrt{(-1-3)^2 + (2+1)^2} = 5$ l.u

$MB = \sqrt{(-1+4)^2 + (2-6)^2} = 5$ l.u

$MC = \sqrt{(-1-2)^2 + (2+2)^2} = 5$ l.u

∴ MA = MB = MC = 5

∴ A, B and C lie on a circle at centre M



circumference $= 2\pi r$

$= 2 \times 3.14 \times 5$

$= 31.4$ length unit

(35) Find the value of a if the distance between (a, 7), (3a − 1, −5) is 13

Sol

$\sqrt{(3a-1-a)^2 + (-5-7)^2} = 13$

$\sqrt{(2a-1)^2 + 144} = 13$

by squaring

$(2a-1)^2 + 144 = 169$

$(2a-1)^2 = 169 - 144$

$(2a-1)^2 = 25$

$2a - 1 = \pm 5$

$2a - 1 = 5$	$2a - 1 = -5$
$2a = 5 + 1$	$2a = -5 + 1$
$2a = 6$	$2a = -4$
$a = 3$	$a = -2$

(36) if A(x, 3), B(3, 2), C(5, 1) if AB = BC find x

Sol

AB = BC

$\sqrt{(x-3)^2 + (3-2)^2} = \sqrt{(3-5)^2 + (2-1)^2}$

$\sqrt{(x-3)^2 + 1} = \sqrt{5}$

by squaring

$(x-3)^2 + 1 = 5$

$(x-3)^2 = 5 - 1$

$(x-3)^2 = 4$

$x - 3 = \pm 2$

$x - 3 = 2$	$x - 3 = -2$
$x = 3 + 2 = 5$	$x = -2 + 3 = 1$

(37) if $C(6,-4)$ is the midpoint of $\overline{AB}$ where $A(5,-3)$ find the coordinates of B

Sol

A(5,-3) C(6,-4) B(x,y)

$\frac{5+x}{2} = \frac{6}{1}$

$5+x = 12$

$x = 12-5 = 7$

$\frac{-3+y}{2} = \frac{-4}{1}$

$-3+y = -8$

$y = -8+3$

$y = -5$

$B(7,-5)$

E is midpoint of $\overline{AC}$

$= \left(\frac{3+1}{2}, \frac{-1+7}{2}\right)$

$= (2,3)$

$\frac{x+6}{2} = \frac{2}{1}$

$x+6 = 4$

$x = 4-6 = -2$

$\frac{y+2}{2} = \frac{3}{1}$

$y+2 = 6$

$y = 6-2 = 4$

$D(-2,4)$

$DE = \sqrt{(-2-2)^2 + (4-3)^2}$

$= \sqrt{17}$ length unit

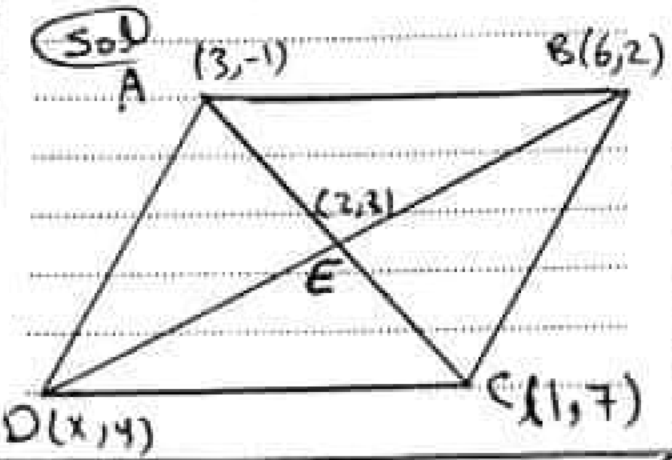
(38) ABCD is a parallelogram its two diagonal intersects at E where $A(3,-1)$, $B(6,2)$, $C(1,7)$

Find

(1) coordinates of E and D

(2) the length of DE

Sol



(39) $\overline{AB}$ is a diameter in a circle M if $B(8,11)$ $M(5,7)$ Find

(1) the coordinates of A

(2) the radius length

(3) the equation of the perpendicular straight line to $\overline{AB}$ at B

Sol

A(x,y) M(5,7) B(8,11)

$\frac{x+8}{2} = 5$

$x+8 = 10$

$x = 2$

$\frac{y+11}{2} = 7$

$y+11 = 14$

$y = 3$

$A(2,3)$

(2) $r = MB = \sqrt{(8-5)^2 + (11-7)^2} = 5$ length unit

(3) Slope of $\overline{AB} = \frac{11-3}{8-2}$

$= \frac{4}{3}$

Slope of perpendicular $= \frac{-3}{4}$

$y = mx + c$

$y = \frac{-3}{4}x + c$

B (8, 11) satisfies the equation

$11 = \frac{-3}{4} \times 8 + c$

$11 = -6 + c$

$11 + 6 = c$

$c = 17$

$y = \frac{-3}{4}x + 17$

(40) Prove that the straight line passes through (−3, −2) and (4, 5) is parallel to the straight line which make with the positive direction of x-axis an angle of measure 45°

sol $m_1 = \frac{5+2}{4+3} = 1$

$m_2 = \tan 45 = 1$

$\because m_1 = m_2$

$\therefore l_1 // l_2$

(41) Prove that the straight line passes through $(4, 3\sqrt{3})$ and $(5, 2\sqrt{3})$ is perpendicular to the straight line which make an angle of measure 30° with the positive direction of x-axis

$m_1 = \frac{2\sqrt{3} - 3\sqrt{3}}{5-4} = -\sqrt{3}$

$m_2 = \tan 30 = \frac{1}{\sqrt{3}}$

$m_1 \times m_2 = -\sqrt{3} \times \frac{1}{\sqrt{3}} = -1$

$\therefore l_1 \perp l_2$

(42) Prove that the points A(0, 2), B(1, 5), C(2, 8) are collinear

sol Slope of $\overleftrightarrow{AB} = \frac{5-2}{1-0} = 3$

Slope of $\overleftrightarrow{BC} = \frac{8-5}{2-1} = 3$

Slope of $\overleftrightarrow{AB}$ = Slope of $\overleftrightarrow{BC}$

and B is a common point

$\therefore$ A, B and C are collinear

43 if the points $(0,1)$, $(a,3)$ and $(2,5)$ are collinear, find a

sol ∵ Points are collinear

$m_1 = m_2$

$$\frac{3-1}{a-0} = \frac{5-1}{2-0}$$

$$\frac{2}{a} = \frac{4}{2}$$

$$a = \frac{2\times 2}{4} = 1$$

44 find the measure of the positive angle which the straight line

$3x + 3y + 7 = 0$

makes with the positive direction of x-axis

sol $m = \frac{-3}{3} = -1$

∴ $\tan\theta = -1$

$\theta = 135^\circ$

45 find the length of the intercepted part at y-axis and the slope of the straight line

$$\frac{x}{2} + \frac{y}{3} = 1$$

sol

$$\frac{1}{2}x + \frac{1}{3}y - 1 = 0$$

$$m = \frac{-\frac{1}{2}}{\frac{1}{3}} = \frac{-3}{2}$$

$$C = \left|\frac{1}{\frac{1}{3}}\right| = 3 \text{ l.u}$$

46 if Δ xyz is right at y, $x(3,5)$, $y(4,2)$, $z(3,a)$ find a

sol ∵ Δxyz right at y

∴ $\overleftrightarrow{xy} \perp yz$

$m_1 \times m_2 = -1$

$$\frac{2-5}{4-3} \times \frac{a-2}{3-4} = -1$$

$$\frac{-3}{1} \times \frac{a-2}{-1} = -1$$

$-3a + 6 = 1$

$-3a = 1 - 6$

$+3a = +5$

∴ $a = \frac{5}{3}$

(47) If the equations

$2x - 3y + a = 0$

$3x + by - 6 = 0$

are equations of two straight lines

find b

(1) if $L_1 // L_2$

(Sol) $m_1 = m_2$

$\frac{-2}{-3} = \frac{-3}{b}$

$b = \frac{-3 \times 3}{2} = \frac{-9}{2}$

[2] $L_1 \perp L_2$

$\therefore m_1 \times m_2 = -1$

$\frac{2}{3} \times \frac{-3}{b} = -1$

$\frac{-6}{3b} = \frac{-1}{1}$

$-3b = -6$

$b = 2$

[3] if (1, 3) lies on L_1 find a

$(\overset{x}{1}, \overset{y}{3})$ satisfy the equation of L_1

$2 \times 1 - 3 \times 3 + a = 0$

$2 - 9 + a = 0$

$a = 7$

[48] Find the equation of the straight line its slope $\frac{1}{2}$ and the intercepts part of y axis 2 length unit in the positive part

find the intersection point with x-axis and y-axis

(Sol) $y = mx + c$

$y = \frac{1}{2}x + 2$

the intersection point with

y-axis (0, 2)

x-axis = (-4, 0)

(49) find the equation of the straight line passes through the points (2,3) and (−3,2)

(sol)

$m = \frac{2-3}{-3-2} = \frac{1}{5}$

$y = \frac{1}{5}x + c$

$(\overset{x}{2},\overset{y}{3})$ satisfies the equation

$3 = \frac{1}{5} \times 2 + c$

$3 = \frac{2}{5} + c$

$c = 3 - \frac{2}{5} = \frac{13}{5}$

$y = \frac{1}{5}x + \frac{13}{5}$

(50) find the equation of the straight line passes through (3,4) and perpindicular to $5x - 2y + 7 = 0$

(sol) $m_{given} = \frac{-5}{-2} = \frac{5}{2}$

$m_{perpindicular} = \frac{-2}{5}$

$y = \frac{-2}{5}x + c$

(3,4) satisfies the equation

$4 = -\frac{2}{5} * 3 + c$

$c = \frac{6}{5} + 4 = \frac{26}{5}$

$\therefore y = -\frac{2}{5}x + \frac{26}{5}$

(51) find the equation of the straight line passes through (1,6) and the mid point of $\overline{AB}$ where A(1,−2) B(3,−4)

(sol) mid of $\overline{AB}$ $(\frac{1+3}{2}, \frac{-2+-4}{2})$

$= (2, -3)$

line passes through (1,6), (2,−3)

$m = \frac{-3-6}{2-1} = -9$

$y = -9x + c$

(1,6) satisfies

$6 = -9 \times 1 + c \quad c = 15$

$y = -9x + 15$

(52) find the equation of the straight line which cuts two positive parts of x-axis and y-axis respectively 4 and 9

(sol) the straight line passes through (4, 0) and (0, 9)

$m = \frac{9-0}{0-4} = \frac{-9}{4}$

$y = mx + c$

$y = \frac{-9}{4} + 9$

(53) from the following table

x	1	2	3
y	1	3	a

find the equation of the straight line and find a

(sol) the line passes through (1,1), (2,3)

$m = \frac{3-1}{2-1} = 2$

$y = 2x + c$

(1,1) satisfies equation

$1 = 2 \cdot \times 1 + c$

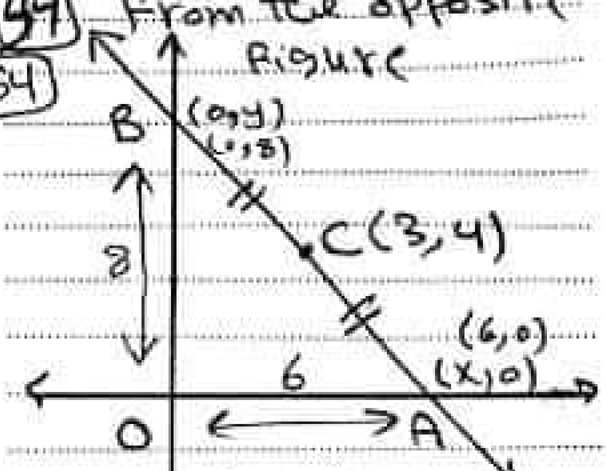
$c = -1$

$y = 2x - 1$

(3,a) satisfies

$a = 2\times3 - 1 = 5$

(54) from the opposite figure



① find A and B
② equation of $\overleftrightarrow{AB}$
③ area of $\triangle ABO$

(sol) $\frac{x+0}{2} = 3 \Rightarrow x = 6$

$\frac{y+0}{2} = 4 \Rightarrow y = 8$

Slope of $\overleftrightarrow{AB} = \frac{0-8}{6-0} = \frac{-4}{3}$

$y = \frac{-4}{3}x + 8$ equation of AB

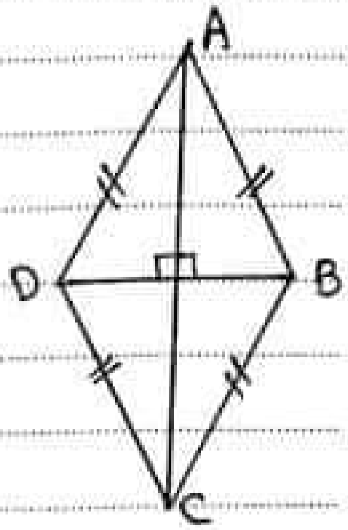
$y = \frac{-4}{3}x + 8$

area of $\triangle ABO$

$= \frac{1}{2} \times 8 \times 6 = 24$ squared unit

(55) Prove that the points A(5,3), B(6,-2), C(1,-1), D(0,4) are a vertices of rhombus then find its area

Sol



$AB = \sqrt{(5-6)^2 + (3+2)^2} = \sqrt{26}$ L.u

$BC = \sqrt{(6-1)^2 + (-2+1)^2} = \sqrt{26}$ L.u

$CD = \sqrt{(0-1)^2 + (4+1)^2} = \sqrt{26}$ L.u

$DA = \sqrt{(0-5)^2 + (4-3)^2} = \sqrt{26}$ L.u

diagonals

$AC = \sqrt{(5-1)^2 + (3+1)^2} = 4\sqrt{2}$ L.u

$BD = \sqrt{(6-0)^2 + (-2-4)^2} = 6\sqrt{2}$ L.u

$AB = BC = CD = DA$

$AC \neq BD$

$\therefore$ ABCD is a rhombus

$\text{Area} = \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ squared unit

(56) Prove that the points A(-2,5), B(3,3), C(-4,2) are not collinear. If D(-9,4) prove that ABCD is a parallelogram

Sol

slope of $\overleftrightarrow{AB} = \frac{3-5}{3-(-2)} = \frac{-2}{5}$

slope of $\overleftrightarrow{BC} = \frac{2-3}{-4-3} = \frac{1}{7}$

slope of $\overleftrightarrow{AB} \neq$ slope of $\overleftrightarrow{BC}$

$\therefore$ A, B, C are not collinear

slope of $\overleftrightarrow{CD} = \frac{4-2}{-9+4} = \frac{-2}{5}$

slope of $\overleftrightarrow{DA} = \frac{4-5}{-9+2} = \frac{1}{7}$

$\therefore \overline{AB} \parallel \overline{CD}$

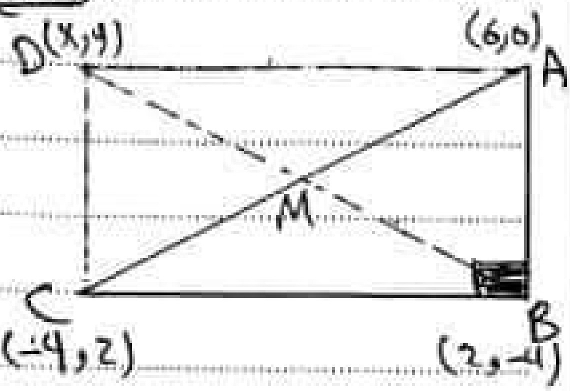
$\overline{BC} \parallel \overline{DA}$

$\therefore$ each two opposite sides are parallel

$\therefore$ ABCD is a parallelogram

(57) Prove that the points A(6, 0), B(2, −4), C(−4, 2) are a vertices of a right angled triangle at B. Find the coordinates of D which make ABCD is a rectangle.

Sol



$AB = \sqrt{(6-2)^2 + (0+4)^2} = 4\sqrt{2}$

$BC = \sqrt{(2+4)^2 + (-4-2)^2} = 6\sqrt{2}$

$AC = \sqrt{(6+4)^2 + (0-2)^2} = \sqrt{104}$

$(AB)^2 = (4\sqrt{2})^2 = 32$

$(BC)^2 = (6\sqrt{2})^2 = 72$

$(AC)^2 = (\sqrt{104})^2 = 104$

$(AC)^2 = (AB)^2 + (BC)^2$

∴ ABC is right angled triangle at B

let M is midpoint of $\overline{AC}$

$M = \left(\frac{6+(-4)}{2}, \frac{0+2}{2}\right) = (1, 1)$

M is midpoint of $\overline{BD}$

$M = \left(\frac{x+2}{2}, \frac{y+(-4)}{2}\right)$

$\therefore \frac{x+2}{2} = \frac{1}{1}$ | $\frac{y-4}{2} = \frac{1}{1}$

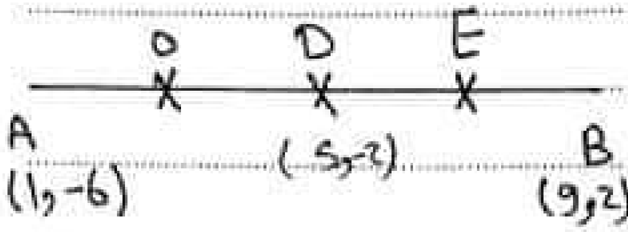
$x+2 = 2$ | $y-4 = 2$

$x = 2-2 = 0$ | $y = 2+4 = 6$

$\therefore D(0, 6)$

(58) If A(1, −6), B(9, 2) find the coordinates of the points which divides $\overline{AB}$ into four equal parts.

Sol



D is midpoint of $\overline{AB}$

$= \left(\frac{1+9}{2}, \frac{-6+2}{2}\right)$

$= (5, -2)$

E is midpoint of $\overline{DB}$

$= \left(\frac{5+9}{2}, \frac{-2+2}{2}\right) = (7, 0)$

O is midpoint of $\overline{AD}$

$= \left(\frac{1+5}{2}, \frac{-6+(-2)}{2}\right)$

$= (3, -4)$

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(59) prove that the straight line passes through (2, 5), (4, 5) is perpendicular to the straight line which passes through (3, 7) and (3, 9)

sol

$m_1 = \frac{5-5}{4-2} = \frac{0}{2} = 0$

$L_1 \parallel$ x-axis

$m_2 = \frac{9-7}{3-3} = \frac{2}{0}$ undefined

$L_2 \parallel$ y-axis

$\therefore L_1 \perp L_2$

(60) if $\overleftrightarrow{CD} \parallel$ x-axis, C(4, 2), D(−5, y) find y

sol $\because \overleftrightarrow{CD} \parallel$ x-axis

$\therefore y_1 = y_2$

$\therefore y = 2$

(61) if $\overleftrightarrow{AB} \parallel$ y-axis, A(x, 7), B(3, 5) find x

sol $\because \overleftrightarrow{AB} \parallel$ y-axis

$\therefore x_1 = x_2 \Rightarrow x = 3$

(62) find the equation of the straight line which passes through (3, 4) and parallel to $x - 3y + 5 = 0$

sol

$m_{given} = \frac{-1}{-3} = \frac{1}{3}$

$m_{parallel} = \frac{1}{3}$

$y = \frac{1}{3}x + c$

(3, 4) satisfy the equation

$\therefore 4 = \frac{1}{3} \times 3 + c$

$4 = 1 + c$

$4 - 1 = c \quad c = 3$

$y = \frac{1}{3}x + 3$

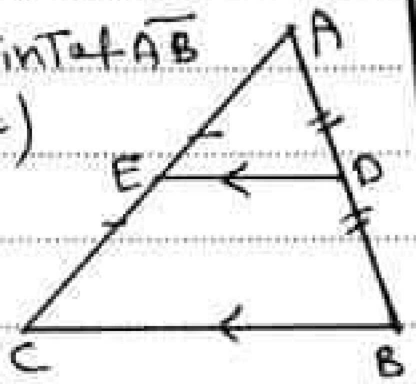
(63) ABC is a triangle A(1, 2), B(5, −2), C(3, 4). D is midpoint of $\overline{AB}$, $\overrightarrow{DE}$ is drawn parallel to $\overline{BC}$ and cuts $\overline{AC}$ at E. find the equation of $\overrightarrow{DE}$

sol

D is midpoint of $\overline{AB}$

$= \left(\frac{1+5}{2}, \frac{-2+2}{2}\right)$

$= (3, 0)$



E is midpoint of $\overline{AC}$

$= \left(\frac{1+3}{2}, \frac{2+4}{2}\right)$

$= (2,3)$

Slope of $\overleftrightarrow{DE} = \frac{3-0}{2-3}$

$= -3$

equation of $\overleftrightarrow{DE}$

$y = -3x + C$

$(2,3)$ satisfy the equation

$3 = -3 \times 2 + C$

$3 = -6 + C$

$3 + 6 = C$

$C = 9$

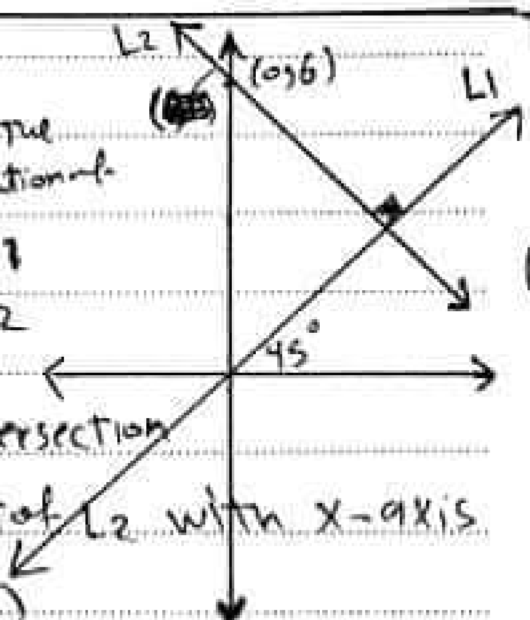
$y = -3x + 9$

(64) Find the equation of

① L_1

② L_2

③ intersection point of L_2 with x-axis



Sol

To find equation of L_1

slope of L_1, $m_1 = \tan 45 = 1$

L_1 passes through the origin

$y = x$ equation of L_1

$L_2 \perp L_1$

slope of L_2 $m_2 = -1$

$y = -x + c$

L_2 cut y-axis at $(0,6)$

$y = -x + 6$ equation of L_2

L_2 cuts x-axis at

$\left(\frac{-6}{-1}, 0\right) = (6,0)$

(65) Find the equation of the straight line passes through $(3,-4)$ and parallel to x-axis

Sol $y = y_1$ $y = -4$

(66) The equation of the straight line passes through $(5,4)$ and parallel to y-axis

Sol $x = x_1$ $x = 5$

(67) Find the intersection point of $2x - 3y + 6 = 0$ with the two axes

Sol the straight line cuts y-axis at $\left(0, \frac{-6}{-3}\right)$

$= (0,2)$

cuts x-axis at $\left(\frac{-6}{2}, 0\right)$

$(-3,0)$

Cumulative Problems from the previous Years

Complete

1. the sum of measures of the accumulative angles at a point = ...

2. The sum of measures of the interior angles of the hexagon ....

3. the number of diagonals of the pentagon = ... and of hexagon =

4. ΔABC in which $m(\hat{B}) = 3m(\hat{A}) = 90°$ then $m(\hat{C}) =$

5. if ABCD is a parallelogram $m(\hat{A}) : m(\hat{B}) = 1 : 3$ then $m(\hat{B}) =$

6. if 3, 7, K are lengths of triangle then K may be = (1, 3, 4, 7)

7. the number of axes of symmetry of the isosceles triangle = ... and of the equilateral triangle = ...

8. the two base angle of the isosceles triangle are ....

9. in ΔABC $m(\hat{B}) > m(\hat{C})$ then AB .... AC

10. The longest side in the right angled triangle is ....

11. the quadrilateral whose diagonal are equal in length and perpindicular is ....

12. The measure of the exterior angle at any vertex of the equilateral triangle = ...

(13) If $\overline{AB} \equiv \overline{CD}$
Then AB − CD = ...

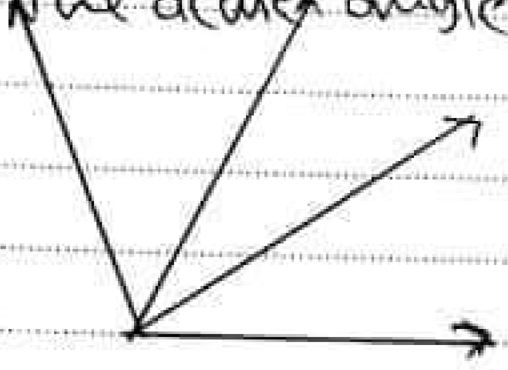
(14) The Image of the point (−3, 5) by reflection in X-axis is ...... and by reflection in y-axis is ...... and in origin point is ......

(15) The Image of the point (2, 4) by a translation (2, 1) is ......

(16) The image of the point (−1, 2) by a rotation about (o) by 180° is ......

(17) the image of the point (1, 2) by a rotation about O by 90° is ......

(18) the number of the acute angle = ...



(19) the sum of measures of the two complementory angles = ... and the sum of measures of two supplementary angles = ...

(20) if $m(\hat{A}) = 100°$ then m(reflex $\hat{A}$) = ...

(21) if two straight line intersects then each two vertically opposite angles are ......

(22) in $\triangle ABC$ if AB = AC = BC then $m(\hat{A})$ = ...

(23) If $\triangle ABC \sim \triangle XYZ$ then $m(\hat{A}) = m(\hat{\ })$

(24) the point of concurence of medians of triangle divides it in the ratio ...... from the side of base and in the ratio ...... from the side of vertex

Mr. HASSAN SALAMA 01141060113 / 01283914718

(25) area of the circle = ----- and its circumference = -----

(26) The triangle whose side lengths 5, 5 cm, ----- is isosceles triangle (9, 10, 11, 12)

(27) △ABC AB > AC then m(B̂) --- m(Ĉ)

(28) The sum of measures of the interior angles of the triangle = ---

(29) The perpendicular straight line to a line segment from its mid point is called -----

(30) in the right angled triangle the length of the opposite side to an angle of measure = 30° = --- length of the hypotenuse

(31) the circumference of a circle whose diameter length = 14 cm = ----- cm

(32) ABCD is a parallelogram then m(Â) + m(Ĉ) = 200 then m(B̂) = ---

(33) the rhombus whose diagonal lengths 6 cm and 8 cm its area = ----- cm²

(34) a square whose diagonal length 10 cm its area = --- cm²

(35) the two parallel straight lines to a third are -----

(36) If ABCD is a square then m(CÂB) = ---

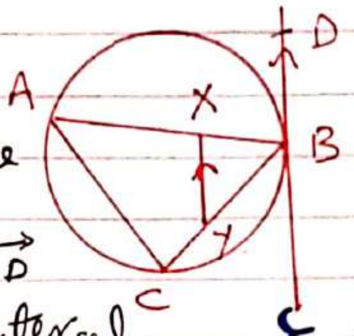
(37) the two perpendicular lines to a third are -----

(38) The median of triangle divides its surface into two triangles -----

(39) The parallelograms whose diagonals equal and not perpendicular is ---

Final revision on geometry 3rd prep

In The opposite figure:



ABC is a triangle inscribed in a circle, $\overleftrightarrow{BD}$ is a tangent to the circle at B, $X \in \overline{AB}$, $Y \in \overline{BC}$ where $\overline{XY} \parallel \overline{BD}$

prove that: AXYC is a cyclic quadrilateral.

Solution:

$\because$ $\overleftrightarrow{BD}$ is a tangent to the circle at B

$\therefore$ $m(\angle DBA) = m(\angle C)$ --- (1)

tangency and inscribed angles subtended by $\widehat{AB}$

$\because$ $\overline{XY} \parallel \overleftrightarrow{BD}$ $\therefore$ $m(\angle DBA) = m(\angle BXY)$ --- (2)

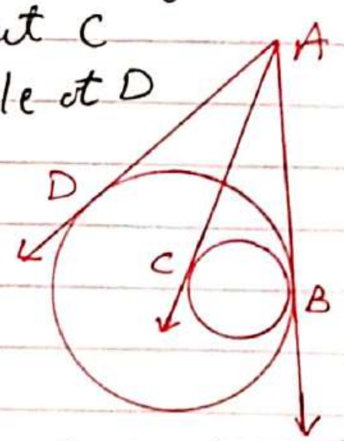
"alternate angles"

from (1) and (2) $\therefore$ $m(\angle BXY) = m(\angle C)$

Exterior and interior angle at the opposite vertex

$\therefore$ AXYC is a cyclic quadrilateral.

In the opposite figure: $\overrightarrow{AB}$ is a common Tangent



$\overrightarrow{AC}$ is a tangent to the smaller circle at C

$\overrightarrow{AD}$ is a tangent to the greater circle at D

$AC = 15$ cm, $AB = (2x - 3)$ cm

and $AD = (y - 2)$ cm.

Find the value of x and y.

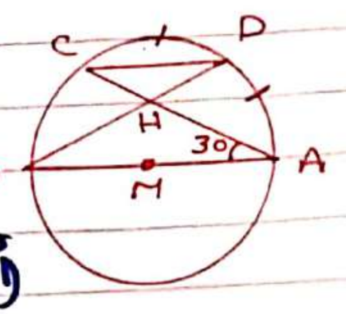
Solution:

$\because$ $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the smaller circle $\therefore$ $AB = AC \Rightarrow 2x - 3 = 15 \Rightarrow$ **$x = 9$**

$\because$ $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are two tangents to the greater circle $\therefore$ $AB = AD \Rightarrow y - 2 = 15 \Rightarrow$ **$y = 17$**

In the opposite figure:

$\overline{AB}$ is a diameter in the circle M, C ∈ the circle M, $m(\angle CAB) = 30°$, D is midpoint of $\widehat{AC}$, $\overline{DB} \cap \overline{AC} = \{H\}$



[1] Find: $m(\angle BDC)$ and $m(\widehat{AD})$ أ/ أحمد عمر

[2] prove that: $\overline{AB} \parallel \overline{DC}$

Solution:

$m(\angle BDC) = m(\angle BAC) = 30°$ (first)

two inscribed angles subtended by $\widehat{BC}$

$\therefore m(\widehat{BC}) = 60°$

$, \because \overline{AB}$ is a diameter $\therefore m(\widehat{AB}) = 180°$

$\because$ D is midpoint of $\widehat{AC}$

$\therefore m(\widehat{AD}) = m(\widehat{DC}) = \frac{180 - 60}{2} = \frac{120}{2} = 60°$

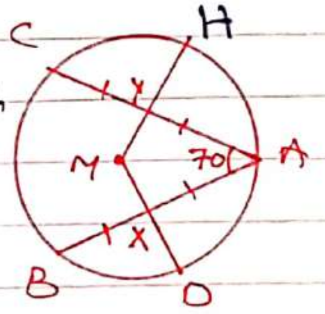
$\because m(\angle DCA) = \frac{1}{2} m(\widehat{AD}) = 30$

$\therefore m(\angle BAC) = m(\angle DCA)$ and they are alternate

$\therefore \overline{AB} \parallel \overline{DC}$

In the opposite figure:

$\overline{AB}$ and $\overline{AC}$ are two equal chords in length in circle M, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$, $m(\angle CAB) = 70°$



[1] Calculate: $m(\angle DMH)$ [2] prove that: $XD = YH$

Solution:

$\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$ --- ①

$\because$ Y is the midpoint of $\overline{AC}$ $\therefore \overline{MY} \perp \overline{AC}$ --- ②

$\therefore AB = AC$ --- ③

$\because$ the sum of interior angles of quadrilateral $= 360$

$\therefore m(\angle DMH) = 360 - (90 + 90 + 70) = 110°$

from ①, ② and ③ $MX = MY$ -- ④ $\because MD = MH = r$ -- ⑤

by subtracting ④ from ⑤ $\therefore XD = YH$

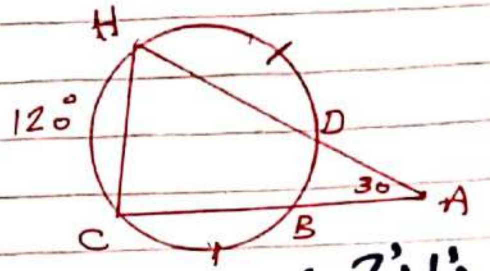
In the opposite figure:

$m(\angle A) = 30^\circ$, $m(\overset{\frown}{HC}) = 120^\circ$.

$m(\overset{\frown}{BC}) = m(\overset{\frown}{DH})$

① find: $m(\overset{\frown}{BD}$ the minor$)$

② prove that: $AB = AD$



Solution:

$$m(\angle A) = \frac{1}{2}[m(\overset{\frown}{HC}) - m(\overset{\frown}{DB})]$$

$$30 = \frac{1}{2}[120 - m(\overset{\frown}{BD})] \quad \times 2$$

$$60 = 120 - m(\overset{\frown}{BD}) \Rightarrow \boxed{m(\overset{\frown}{BD}) = 60^\circ}$$

$\because m(\overset{\frown}{HD}) = m(\overset{\frown}{CB}) \quad \therefore HD = CB \rightarrow$ ①

and $m(\overset{\frown}{HD}) + m(\overset{\frown}{DB}) = m(\overset{\frown}{CB}) + m(\overset{\frown}{DB})$

$\therefore m(\overset{\frown}{HDB}) = m(\overset{\frown}{CBD}) \Rightarrow m(\angle C) = m(\angle H)$

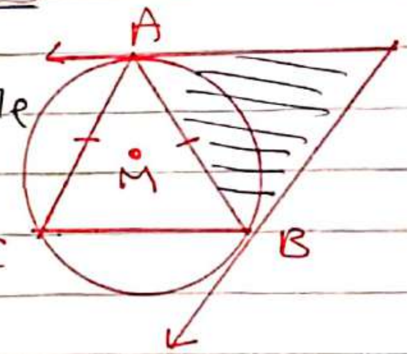
$\therefore AH = AC \rightarrow$ ②

by Subtracting ① from ② $\quad \therefore AB = AD$

In the opposite figure:

$\overrightarrow{DA}$ and $\overrightarrow{DB}$ are Two tangents of the Circle M and $AB = AC$ prove that:

$\overrightarrow{AC}$ is a Tangent to the Circle passing through the vertices of $\triangle ABD$



Solution:

hint we need to prove that $m(\angle BAC) = m(\angle$

Solution $\because \overrightarrow{DA}$ is tangent to the circle at A

$\therefore m(\angle DAB)$ Tangency $= m(\angle ACB)$ inscribed --- ①

$\because AC = AB \quad \therefore m(\angle ACB) = m(\angle ABC) \rightarrow$ ②

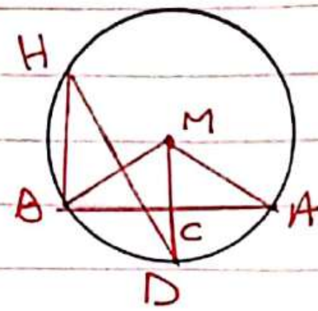
$\because \overrightarrow{DA}$ and $\overrightarrow{DB}$ are Two tangents $\therefore DA = DB$

$\therefore m(\angle DAB) = m(\angle DBA)$ --- ③

from ① and ② and ③ $\therefore m(\angle CAB) = m(\angle D)$

$\therefore \overrightarrow{AC}$ is a tangent ...

In the opposite figure:
C is the midpoint of $\overline{AB}$,
$\overrightarrow{MC} \cap$ the Circle $M = \{D\}$,
$m(\angle MAB) = 20^\circ$
Find: $m(\angle BHD)$ and $m(\overset{\frown}{ADB})$



Solution:

$\because MA = MB = r$

$\therefore m(\angle A) = m(\angle MBA) = 20^\circ$

$\because$ C is the midpoint of $\overline{AB}$ $\quad \therefore \overline{MC} \perp \overline{AB}$

In $\triangle MBC$

$m(\angle BMC) = 180 - (90 + 20) = 70^\circ$

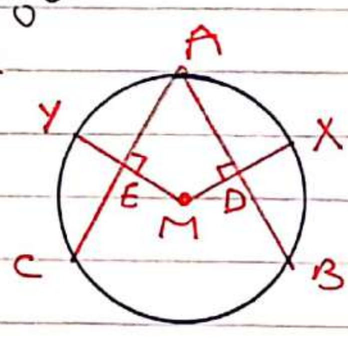
$\Rightarrow \therefore m(\angle AMB) = \frac{1}{2} m(\angle BMD) = 35^\circ$

inscribed angle and central angle subtended by $\overset{\frown}{BD}$

$m(\angle AMB) = 180 - (20 + 20) = 140^\circ$

$\therefore m(\overset{\frown}{ADB}) = m(\angle AMB) = 140^\circ$

In the opposite figure:
$AB = AC$, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$
prove that: $XD = YE$



Solution

$\because \overline{MD} \perp \overline{AB}$
$, \overline{ME} \perp \overline{AC}$
and $AB = AC$
$\}$ $\therefore ME = MD$ --- (1)

$\because MY = MX = r$ --- (2)

by subtracting (1) from (2)

$\therefore YE = XD$ #

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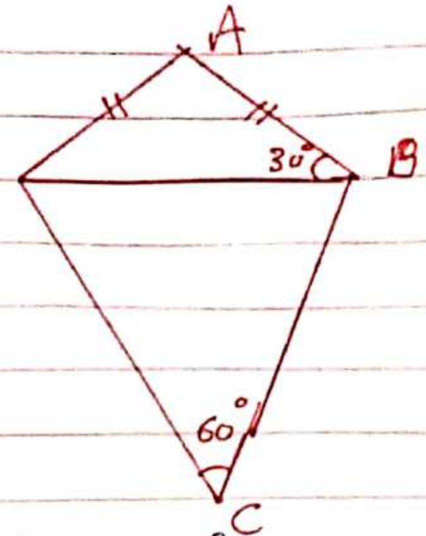

In The opposite figure:

ABCD is a quadrelateral in which

$AB = AD$, $m(\angle ABD) = 30°$, $m(\angle C) = 60°$

prove that: ABCD is a cyclic quad.

Solution:

In Δ ABD $\because$ $AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30$

$\therefore m(\angle DAB) = 180 - (30 + 30) = 180 - 60 = 120°$

$\because m(\angle A) + m(\angle C) = 60 + 120 = 180$

and they are opposite angle

$\therefore$ ABCD is a cyclic quadrelateral.

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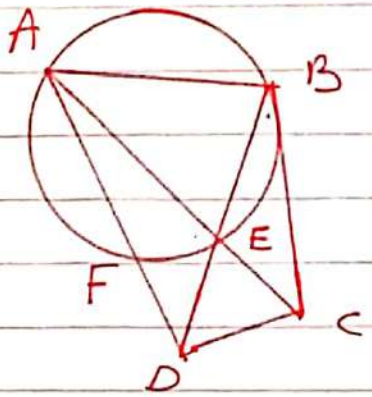


In the opposite figure:

$\overline{BC}$ is a tangent at B;

E is the midpoint of $\overset{\frown}{BF}$

prove that: ABCD is a cyclic quad.

Solution:

$\because$ $\overline{BC}$ is a tangent at B

$\therefore m(\angle CBE)$ tangency $= m(\angle BAE)$ inscribed --- (1)

$\because$ E is the midpoint of $\overset{\frown}{BF}$

$\therefore m(\overset{\frown}{BE}) = m(\overset{\frown}{EF})$

$\therefore m(\angle BAE) = m(\angle EAF)$ --- (2)

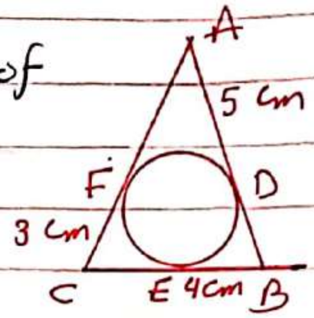
inscribed angles subtended by equal arcs.

from (1) and (2) $\therefore m(\angle CBD) = m(\angle CAD)$

drawn on $\overline{CD}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrelateral.

In the opposite figure:
A circle is drawn touches the sides of a triangle ABC, $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D, E, F, AD = 5 cm, BE = 4 cm, CF = 3 cm



Find: The perimeter of ΔABC

Solution:

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$\because \overrightarrow{AD}, \overrightarrow{AF}$ are two tangent-segments

$\therefore AD = AF = 5$ cm

, $\because \overline{BD}, \overline{BE}$ are two tangent-segments

$\therefore BD = BE = 4$ cm

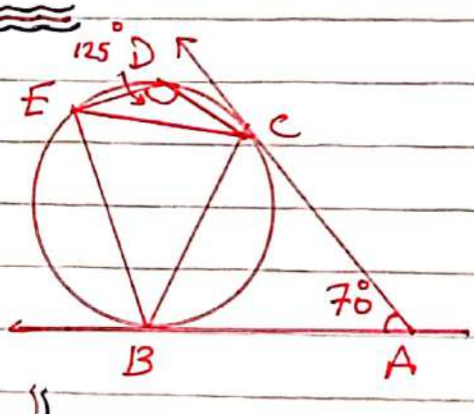
and $\because \overline{CF}, \overline{CE}$ are two Tangent-segments

$\therefore CE = CF = 3$ cm

$\therefore$ perimeter of $\Delta ABC = 5+5+4+4+3+3 = 24$ cm

In the opposite figure:
$\overleftrightarrow{AB}$, $\overrightarrow{AC}$ are two Tangents to the circle at B, C, $m(\angle D) = 125°$, $m(\angle A) = 70°$ prove that:

① $CB = CE$ ② $\overrightarrow{AC} \parallel \overline{BE}$



Solution

$\because \overleftrightarrow{AB}$ and $\overline{AC}$ tangent-segments

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180-70}{2} = 55°$

, $\because m(\angle ACB)$ tangency $= m(\angle BEC) = 55°$ inscribed

$\because EBCD$ is cyclic quad

$\therefore m(\angle B) = 180 - 125 = 55°$

In ΔEBC

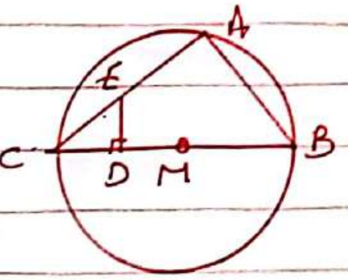
$\because m(\angle EBC) = m(\angle CEB)$

$\therefore EC = BC$ (first)

, $\because m(\angle ACB) = m(\angle CBE)$ and they are alternate

$\therefore \overrightarrow{AC} \parallel \overline{BE}$

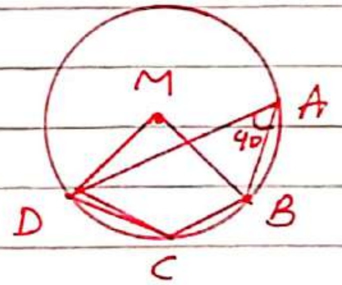
In the opposite figure:
$\overline{BC}$ is a diameter in the circle M
, $\overline{ED} \perp \overline{BC}$
prove that: [1] ABDE is a cyclic quad.
[2] $m(\angle DEC) = \frac{1}{2} m(\widehat{AC})$



Solution:

$\because \overline{BC}$ is a diameter $\therefore m(\angle BAC) = 90^\circ$
inscribed drawn in a semicircle
$\because \overline{ED} \perp \overline{BC}$ $\therefore m(\angle EDB) = 90$ أ/ محمد
$\therefore m(\angle BAE) + m(\angle EBD) = 90 + 90 = 180^\circ$
and they are opposite
$\therefore$ ABDE is a cyclic quadrilateral
$\therefore m(\angle DEC)$ Exterior $= m(\angle B)$ interior
and $\because m(\angle B) = \frac{1}{2} m(\widehat{AC})$
$\therefore m(\angle DEC) = \frac{1}{2} m(\widehat{AC})$

In the opposite figure
Find ① $m(\angle BMD)$ ② $m(\angle BCD)$



Solution:

$m(\angle BMD)$ central $= 2m(\angle BAD)$
$= 80^\circ$ inscribed

$m(\angle BCD)$ inscribed $= \frac{1}{2}$(reflex $\angle BMD$) $= \frac{1}{2} \times 280 = 140^\circ$
central

another solution:
$\because$ ABCD is a cyclic quadrilateral

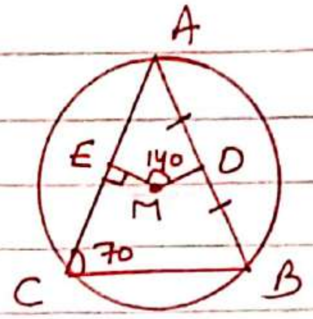
$\therefore m(\angle C) = 180 - 40 = 140^\circ$

In the opposite figure:

D is the midpoint of $\overline{AB}$, $\overline{ME} \perp \overline{AC}$

$m(\angle DME) = 140°$ and $m(\angle C) = 70°$

prove that: $MD = ME$



Solution:

$\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

$\because$ the sum of interior angles of the quad. $= 360°$

$\therefore m(\angle A) = 360 - (90 + 90 + 140) = 40°$

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In $\triangle ABC$

$m(\angle B) = 180 - (70 + 40) = 70°$

$\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

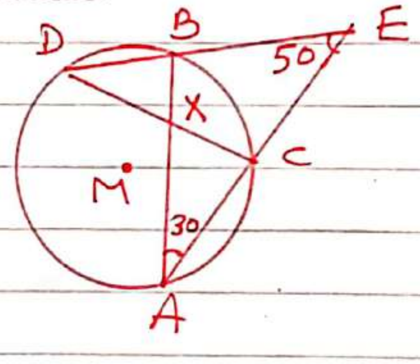
$\because \overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$ and $AB = AC$

$\therefore MD = ME$

In the opposite figure:

find each of ① $m(\widehat{AD})$

② $m(\angle AXD)$



Solution

$m(\widehat{BC}) = 2\, m(\angle A) = 60°$

$\because \overrightarrow{AC} \cap \overrightarrow{DB} = \{E\}$

$\therefore m(\angle E) = \frac{1}{2}[m(\widehat{AD}) - m(\widehat{BC})]$

$\therefore 50 = \frac{1}{2}[m(\widehat{AD}) - 60]$ ×2

$100 = m(\widehat{AD}) - 60$

$\Rightarrow m(\widehat{AD}) = 100 + 60 = 160°$ "first"

$\because \overline{DC} \cap \overline{AB} = \{X\}$

$\therefore m(\angle AXD) = \frac{1}{2}[m(\widehat{AD}) + (\widehat{BC})]$

$= \frac{1}{2}[160 + 60] = 110°$

"second"

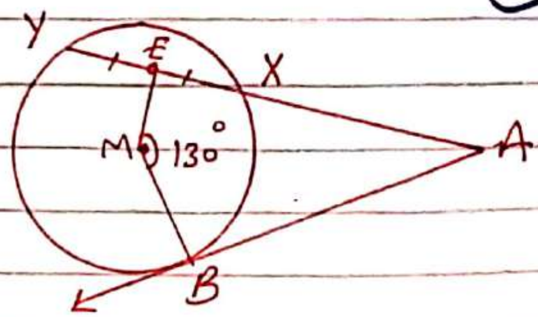
In the opposite figure:

$\overrightarrow{AB}$ is a tangent to the circle M

, E is midpoint of $\overline{XY}$

$m(\angle BME) = 130°$

Find , $m(\angle A)$



Solution:

$\because \overrightarrow{AB}$ is a tangent $\therefore \overline{MB} \perp \overrightarrow{AB}$

$\because$ E is the midpoint of $\overline{XY}$ $\therefore \overline{ME} \perp \overline{XY}$

$\because$ the sum of interior angles of quad. $= 360°$

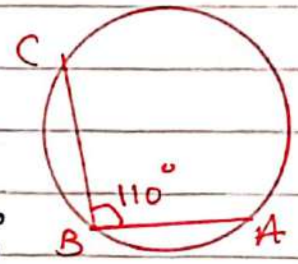
$\therefore m(\angle A) = 360 - (130 + 90 + 90) = 50°$ أ/ أحمد عمر

⇒

In the opposite figure:

$m(\angle ABC) = 110°$

find $m(\widehat{ABC})$



Solution:

$m(\widehat{AC}) = 2m(\angle ABC) = 220°$

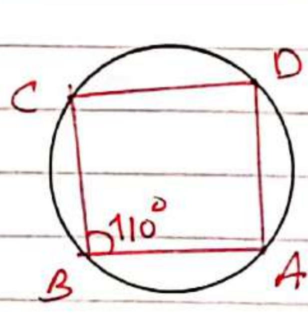
$\because$ the measure of the circle $= 360°$

$\therefore m(\widehat{ABC}) = 360 - 220 = 140°$

another solution:

put D on the circle and

Draw $\overline{DA}$ and $\overline{DC}$



$\because$ ABCD is a cyclic quad.

$\therefore m(\angle D) = 180 - 110 = 70°$

$\therefore m(\widehat{ABC}) = 2m(\angle D) = 140°$

M is a circle with radius length 6.3 Cm

$m(\angle BAC) = 40^\circ$

Find (1) $m(\angle MBC)$

(2) length $(\overset{\frown}{BC})$ where $(\pi \simeq \frac{22}{7})$

Solution:

$m(\angle BMC) = 2\, m(\angle BAC) = 80^\circ$

inscribed angle and Central angle subtended by $\overset{\frown}{BC}$

In $\triangle MBC$

$\because MB = MC = r$

$\therefore m(\angle MBC) = m(\angle MCB) = \frac{180 - 80}{2} = 50^\circ$

length $(\overset{\frown}{BC}) = \frac{m(\overset{\frown}{BC})}{360} \times 2\pi r$

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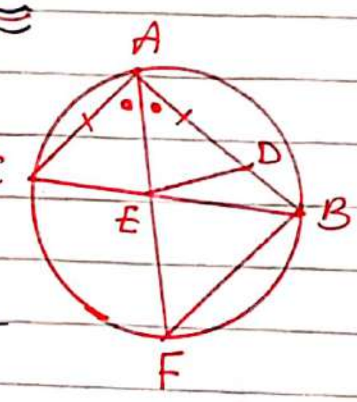
$= \frac{80}{360} \times 2 \times \frac{22}{7} \times 6.3 = 8.8$ cm

---

$AD = AC$, $\overrightarrow{AF}$ bisects $\angle A$

prove that: DBFE is a cyclic quad.



Solution:

In $\triangle\triangle$ ADE, ACE $\begin{cases} AD = AC \\ \overline{AE} \text{ is a common side} \\ m(\angle DAE) = m(\angle CAE) \end{cases}$

$\therefore \triangle ADE \equiv \triangle ACE$

$\therefore m(\angle ADE) = m(\angle ACE)$ --- (1)

$\because m(\angle ACB) = m(\angle AFB)$ --- (2)

two inscribed angles subtended by $\overset{\frown}{AB}$

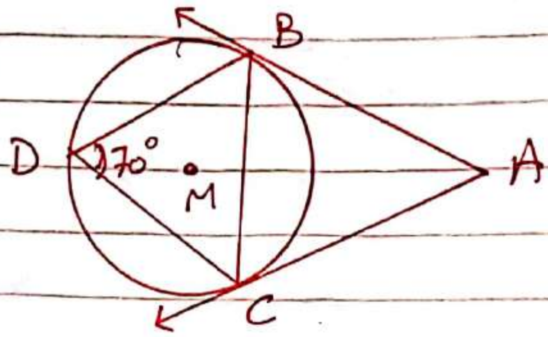
from (1) and (2)

$\therefore m(\angle ADE)$ exterior $= m(\angle EFB)$ interror

$\therefore$ DBFE is a cyclic quad.

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle M, $m(\angle BDC) = 70°$

Find: $m(\angle A)$



Solution:

$\because \overrightarrow{AB}$ is a tangent

$\therefore m(\angle ABC)$ tangency $= m(\angle D)$ inscribed $= 70°$

subtended by the arc $\widehat{BC}$

$\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = 70°$

$\therefore m(\angle A) = 180 - (70 + 70) = 40°$

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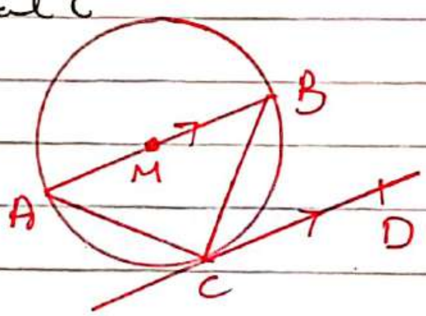
$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{CD}$ is a tangent to the circle at C

, $\overrightarrow{CD} \parallel \overline{AB}$

[1] prove that: $AC = BC$

[2] find: $m(\angle B)$ by degree



Solution:

$\because \overrightarrow{CD}$ is a tangent

and $\overrightarrow{CD} \parallel \overline{AB}$

$\therefore m(\widehat{BC}) = m(\widehat{AC}) \rightarrow (1)$

$\therefore BC = AC$ first

$\because \overline{AB}$ is a diameter

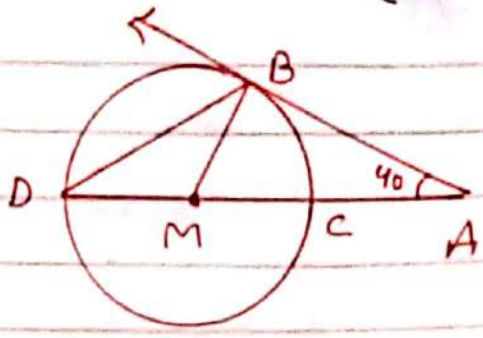
$m(\widehat{BC}) + m(\widehat{AC}) = 180 \rightarrow (2)$

from (1) and (2) $\therefore m(\widehat{AC}) = 90°$

$\therefore m(\angle B)$ inscribed $= \frac{1}{2} m(\widehat{AC}) = 45°$

$\overrightarrow{AB}$ is a tangent to the circle M at B , $m(\angle A) = 40°$

find: $m(\angle BDC)$



Solution:

$\because \overrightarrow{AB}$ is a tangent at B

$\therefore \overline{MB} \perp \overrightarrow{AB}$

In $\triangle AMB$: $m(\angle AMB) = 180 - (90 + 40) = 50°$

$\therefore m(\angle BDC)$ tangency $= \frac{1}{2} m(\angle BMC)$ central

$= 25°$

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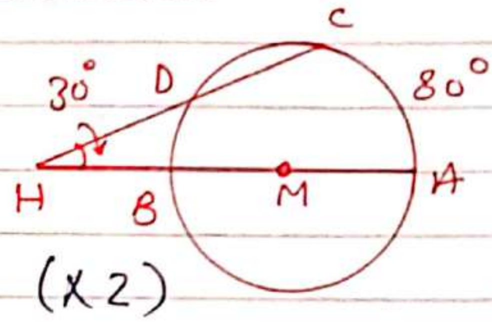
Find: $m(\widehat{CD})$



Solution

$m(\angle H) = \frac{1}{2}[m(\widehat{AC}) - m(\widehat{DB})]$

$\therefore 30 = \frac{1}{2}[80 - m(\widehat{DB})]$ (X2)

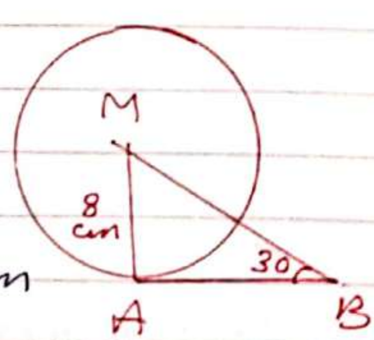
$60 = 80 - m(\widehat{DB}) \Rightarrow m(\widehat{DB}) = 80 - 60 = 20°$

$\because \overline{AB}$ is a diameter

$\therefore m(\widehat{AC}) + m(\widehat{DC}) + m(\widehat{DB}) = 180°$

$\therefore m(\widehat{DC}) = 180 - (80 + 20) = 80°$

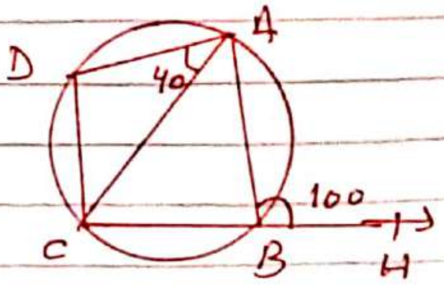
Find: length of $\overline{AB}$



Solution:

$\because \overrightarrow{AB}$ is a tangent $\therefore \overline{MA} \perp \overline{AB}$

$\because \frac{\tan 30}{1} = \frac{8}{AB} \Rightarrow AB = \frac{8}{\tan 30} = 8\sqrt{3}$ cm prove that: AD = DC



Solution:

∵ ABCD is a cyclic quad.

∴ m(ADC) interior = m(∠ABH)
= 100° exterior

In △ADC:

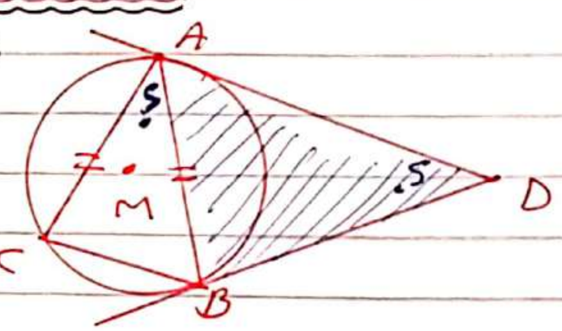
m(∠ACD) = 180 − (100 + 40) = 40°

∵ m(∠DAC) = m(∠DCA) = 40

∴ DC = DA

$\overrightarrow{DA}$ and $\overrightarrow{DB}$ are two tangents to the circle M and AB = AC

prove that: $\overline{AC}$ is a tangent to the circle passing through the vertices of △ABD



Hint: we need to prove that m(∠D) = m(∠CAB)

Solution:

In △ABD

∵ AB = AC ∴ m(∠ACB) = m(∠ABC) --- ①

In △DAB

∵ $\overline{DA}$, $\overline{DB}$ are two tangent-segments

∴ DA = DB

∴ m(DBA) = m(D$\hat{A}$B) --- ②

∵ $\overrightarrow{DA}$ is a tangent

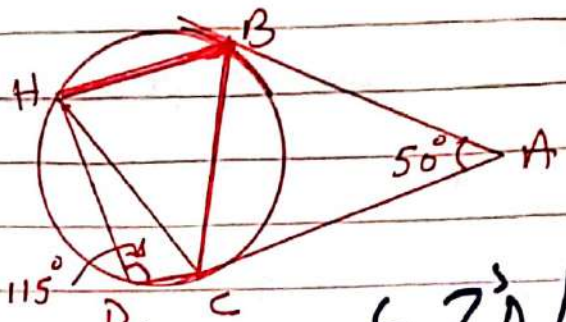
∴ m(∠DAB) tangency = m(∠ACB) inscribed --- ③

from ①, ② and ③ ∴ m(∠CAB) = m(∠D)

∴ $\overline{AC}$ is a tangent to the circle passing through the vertices of △ABD

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the Circle at B, C

prove that: $\overrightarrow{BC}$ bisects $\angle ABH$



Solution:

$\because \overline{AB}, \overline{AC}$ are two tangent-segments

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$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180-50}{2} = \frac{130}{2} = 65°$ ---①

$\because$ DCBH is a cyclic quadrilateral

$\therefore m(\angle HBC) = 180 - 115 = 65°$ ---②

from ① and ②

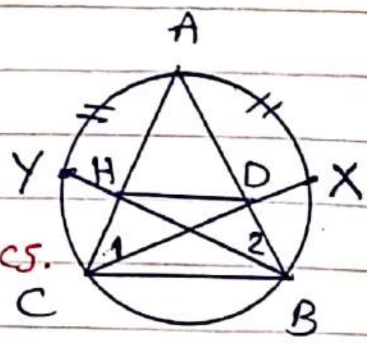
$\therefore m(\angle ABC) = m(\angle HBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle ABH$

ABC is a triangle inscribed in a circle, $X \in \widehat{AB}$, $Y \in \widehat{AC}$, where $m(\widehat{AX}) = m(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$, $\overline{BY} \cap \overline{AC} = \{H\}$

prove that: BCHD is a cyclic quadrilateral.

Solution



$\because m(\widehat{AX}) = m(\widehat{AY})$

$\therefore m(\angle ACX) = m(\angle ABY)$

inscribed angles subtended by equal arcs.

In the quad. BCHD

$\because m(\angle DBH) = m(\angle DCH)$

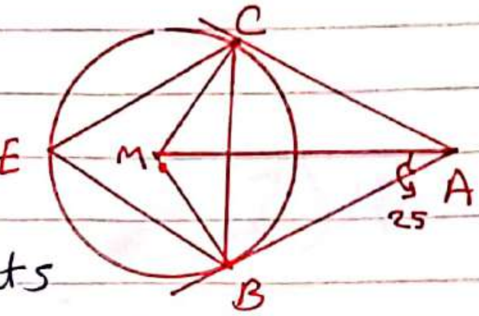
drawn on $\overline{DH}$ and on one side of it

$\therefore$ BCHD a cyclic quad.

$\overline{AB}$ and $\overline{AC}$ are two tangents segments to the circle M, $m(\angle BAM) = 25$

find: [1] $m(\angle ACB)$

[2] $m(\angle BEC)$



Solution:

$\because \overline{AB}$ and $\overline{AC}$ are two tangents

$\therefore \overleftrightarrow{AM}$ bisects $\angle A$ and $\angle CMB$

and $AC = AB$

محمد عمر 24/P

$\therefore m(\angle A) = 2 \times 25 = 50°$

$m(\angle ACB) = (180 - 50) \div 2 = 65°$ (first)

and $\overline{MC} \perp \overline{AC}$, $\overline{MB} \perp \overline{AB}$

$\therefore$ ABMC is a cyclic quadrilateral

$\therefore m(\angle M) + m(\angle A) = 180 \Rightarrow m(\angle CMB) = 180 - 50 = 130°$

$\therefore m(\angle BEC)$ inscribed $= \frac{1}{2} m(\angle CMB)$ central $= 65°$

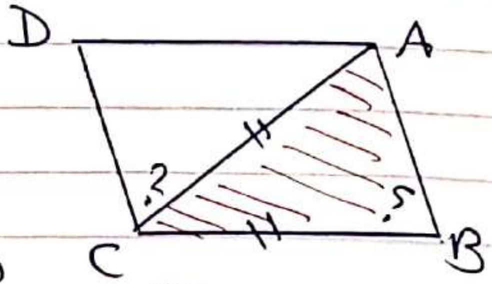
ABCD is a parallelogram in which $AC = AB$

prove that: $\overleftrightarrow{CB}$ is a tangent to the circle circumscribed about the triangle ABC.

Solution:

Hint: we need to prove that

$m(\angle DCA) = m(\angle B)$



Solution:

$\because$ ABCD is a parallelogram

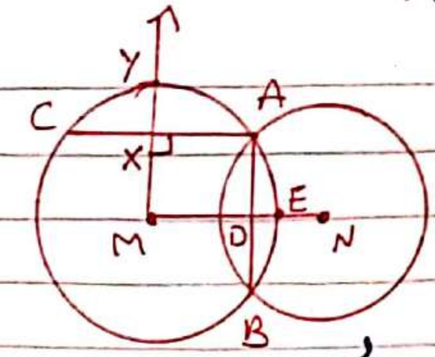
$\therefore m(\angle DCA) = m(\angle CAB)$ "alt." --- ①

In $\triangle ABC$

$\because AC = CB$ $\therefore m(\angle ABC) = m(\angle CAB)$ --- ②

from ① and ② $\therefore m(\angle DCA) = m(\angle ABC)$

$\therefore \overleftrightarrow{CB}$ is a tangent to the circle passes through the vertices of $\triangle ABC$



The two Circles M and N intersect at A and B, $\overline{MX} \perp \overline{AC}$, AC = AB
prove that: XY = DE
Solution

∵ M and N are two Circles intersects at A, B
∴ $\overline{MN} \perp \overline{AB}$ → ①
∵ $\overline{MX} \perp \overline{AC}$ --- ② , AC = AB --- ③
from ①, ② and ③
∴ MX = MD --- ④
∵ MY = ME = r --- ⑤
by subtracting ④ from ⑤ ∴ XY = DE

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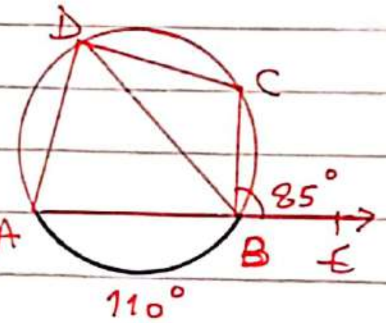
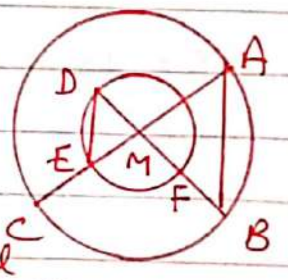


find: $m(\angle BDC)$
Solution: ∵ ABCD is a cyclic quad.
∴ $m(\angle CBE)$ Exterior = $m(\angle D)$ interior
∴ $m(\angle CDA) = 85°$
∵ $m(B\hat{D}A) = \frac{1}{2} m(\widehat{AB}) = \frac{110}{2} = 55°$

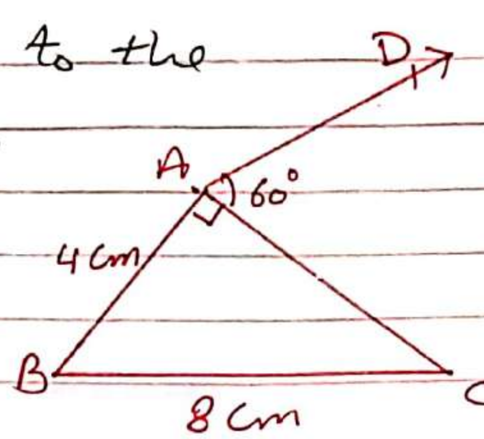
∴ $m(\angle BDC) = 85 - 55 = 30°$



two Concentric Circles with Centre M.
prove that: $m(\angle BAC) = m(\angle FDE)$
Solution
In the smaller circle:
$m(\angle FDE)$ inscribed $= \frac{1}{2} m(\angle FME)$ central --- ①

In the greater Circle
$m(\angle BAC)$ inscribed = $m(\angle BMC)$ central --- ②
from ① and ②
∴ $m(\angle BAC) = m(\angle FDE)$

prove that: $\overrightarrow{AD}$ is a tangent to the Circle which passes through the vertices of $\triangle ABC$



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Solution:

In $\triangle ABC$

$\cos(B) = \frac{4}{8} = \frac{1}{2}$

$\therefore m(\angle B) = 60^\circ$

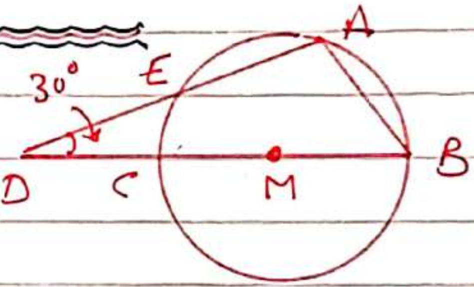
$\because m(\angle DAC) = m(\angle B) = 60$

$\therefore \overrightarrow{AD}$ is a tangent to the circle which passes through the vertices of $\triangle ABC$

$m(\widehat{AEC}) = 100^\circ$, $\overline{BC}$ is a diameter

$m(\angle D) = 30^\circ$

find: $m(\angle BAD)$



Solution:

$\because m(\angle D) = \frac{1}{2}[m(\widehat{AB}) - m(\widehat{EC})]$

, $\because \overline{AB}$ is a diameter $\therefore m(\widehat{BAC}) = 180^\circ$

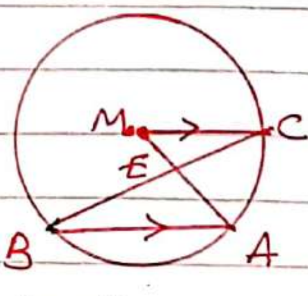
$\therefore m(\widehat{AB}) = 180 - 100 = 80^\circ$

$\therefore 30 = \frac{1}{2}[80 - m(\widehat{EC})]$ (x2)

$80 - m(\widehat{EC}) = 60 \Rightarrow m(\widehat{EC}) = 20^\circ$

$\therefore m(\angle BAD) = m(\angle BAE) = \frac{1}{2} m(\widehat{BCE}) = \frac{1}{2} \times 200 = 100^\circ$

$\overline{CM} \parallel \overline{AB}$ prove that: $BE > AE$



Solution: $\because m(\angle CMA) = 2m(\angle CBA)$

$\therefore m(\angle CMA) > m(\angle CBA)$ --- (1)

$\because \overline{MC} \parallel \overline{BA}$ $\therefore m(\angle CMA) = m(\angle MAB)$ --- (2)

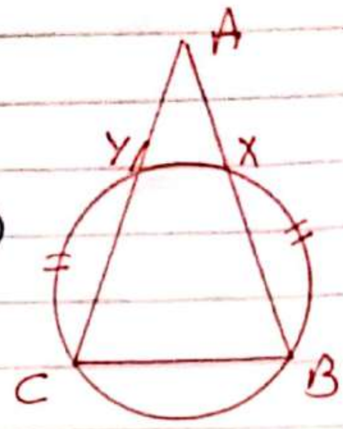
from (1) and (2) $\therefore m(\angle MAB) > m(\angle CBA)$

$\because \triangle EAB$ $\because m(\angle EAB) > m(\angle EBA)$

$\therefore EB > EA$

length of $(\overset{\frown}{BX})$ = length of $(\overset{\frown}{CY})$

prove that: $AX = AY$



Solution

$\because$ length of $(\overset{\frown}{BX})$ = length of $(\overset{\frown}{CY})$

$\therefore m(\overset{\frown}{BX}) = m(\overset{\frown}{CY})$

by adding $m(\overset{\frown}{XY})$ to the two sides

$\therefore m(\overset{\frown}{BXY}) = m(\overset{\frown}{CYX})$

$\therefore m(\angle C) = m(\angle B)$

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inscribed angles subtended by equal arcs

In $\triangle ABC$ $\because m(\angle B) = m(\angle C)$

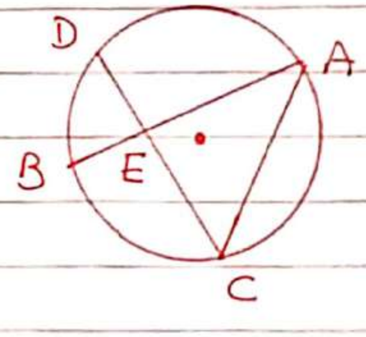
$\therefore AB = AC$ —— ①

$\because m(\overset{\frown}{BX}) = m(\overset{\frown}{CY})$ $\therefore BX = CY$ —— ②

by subtracting ② from ① $\therefore AX = AY$

$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle

prove that: $\triangle ACE$ is an isosceles $\triangle$



Solution:

$\because AB = CD$

$\therefore m(\overset{\frown}{ADB}) = m(\overset{\frown}{CBD})$

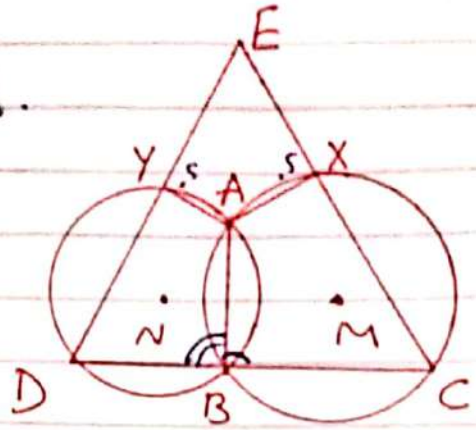
by deleting $m(\overset{\frown}{BD})$ from the two sides

$\therefore m(\overset{\frown}{AD}) = m(\overset{\frown}{BC})$

$\therefore m(\angle C) = m(\angle A)$ inscribed angles subtended by equal arcs.

In $\triangle AEC$ $\because m(\angle A) = m(\angle C)$

$\therefore \triangle AEC$ is an isosceles triangle



prove that: AXEY is a cyclic quad.

proof:

$\because$ AXCB is a cyclic quad.

$\therefore m(\angle AXE)$ exterior $= m(\angle ABC)$ ---> ①

$\because$ ABDY is a cyclic quad.

$\therefore m(\angle AYE)$ exterior $= m(\angle ABD)$ interior ---②

by adding ① and ②

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$\therefore m(\angle AXE) + m(\angle AYE) = m(A\hat{B}C) + m(\angle ABD)$

$= 180^\circ$ "$B \in \overline{CD}$"

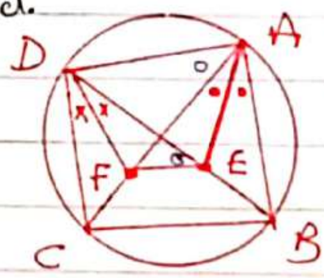
and they are opposite

$\therefore$ AXEY is a cyclic quadrilateral.

---

prove that: ① AEFD is a cyclic quad.

② $\overline{EF} \parallel \overline{BC}$



proof: $\because$ ABCD is a cyclic quad.

$\therefore m(\angle BAC) = m(\angle BDC)$ --- ①

drawn on $\overline{BC}$

$\because \overrightarrow{AE}$ bisects $\angle BAC$ $\therefore m(\angle EAF) = \frac{1}{2} m(\angle BAC)$ --②

$\because \overrightarrow{DF}$ bisects $\angle BDC$ $\therefore m(\angle EDF) = \frac{1}{2} m(\angle BDC)$ -- ③

from ①, ② and ③

$\therefore m(\angle EAF) = m(\angle EDF)$

and they are drawn on one side of $\overline{EF}$

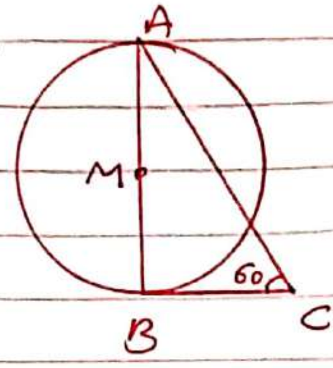
$\therefore$ AEFD is a cyclic quadrilateral.

$\therefore m(\angle DAF) = m(\angle DEF)$ drawn on DF ...④

, $\because m(D\hat{A}C) = m(\angle DBC)$ inscribed sub. by DC ⑤

from ④ and ⑤ $\therefore m(\angle DEF) = m(\angle DBC)$ $\therefore \overline{EF} \parallel \overline{BC}$

the circle M has circumference = 44 cm.
, $\overline{AB}$ is a diameter, $\overrightarrow{BC}$ is tangent at B
$m(\angle C) = 60^\circ$, $\pi \simeq \frac{22}{7}$
find: the length of $\overline{BC}$



solution

$C = 2\pi r$

$\Rightarrow 44 = 2 \times \frac{22}{7} \times r \Rightarrow r = 7$ cm

$\therefore AB = 14$ cm

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$\because$ AB a diameter and $\overrightarrow{BC}$ is a tangent

$\therefore m(\angle B) = 90^\circ$

$\tan C = \frac{opp.}{adj.} = \frac{AB}{BC}$ $\quad \because \frac{\tan 60}{1} = \frac{14}{BC}$

$\therefore BC = \frac{14}{\sqrt{3}} = \frac{14}{3}\sqrt{3} \simeq 8.1$ cm

$\overline{BC}$ is a diameter in the circle M, $\overline{BY}$ is a chord
, $H \in \overrightarrow{BY}$ such that $BY = YH$
prove that: $m(Y\hat{M}C) = 2m(\angle BHC)$
proof

In $\triangle HBC$

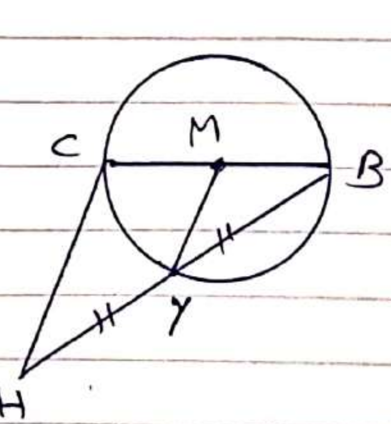
$\because$ M is the midpoint of $\overline{BC}$

and Y is the midpoint of $\overline{BH}$

$\therefore MY = \frac{1}{2} CH \Rightarrow CH = 2MY$

$\therefore CH = 2r = BC$

$\because HC = BC \quad \therefore m(\angle H) = m(\angle B) \dots (1)$



$m(\angle YMC)$ central $= 2\, m(\angle B)$ inscribed $\dots (2)$

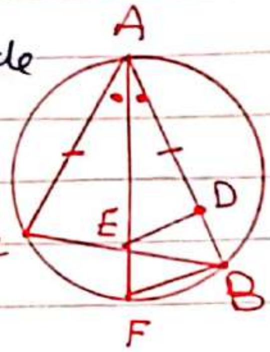
from (1) and (2)

$\therefore m(Y\hat{M}C) = 2m(\angle BHC)$ #

ABC inscribed triangle in a circle such that
$AB > AC$, $D \in \overline{AB}$, $AC = AD$
$\overrightarrow{AE}$ bisects $\angle A$ and intersects $\overline{BC}$ at E and intersects the circle at F

prove that: BDEF is a cyclic quadrilateral.

proof:



In $\Delta\Delta$ ADE, ACE $\begin{cases} AD = AC \\ \overline{AE} \text{ is a common side} \\ m(\angle DAE) = m(\angle CAE) \end{cases}$

$\therefore \Delta ADE \equiv \Delta ACE$

$\therefore m(\angle ACE) = m(\angle ADE)$ --- ①

$\because m(\angle ACD) = m(\angle AFD)$ --- ②
inscribed subtended by $\overset{\frown}{AD}$

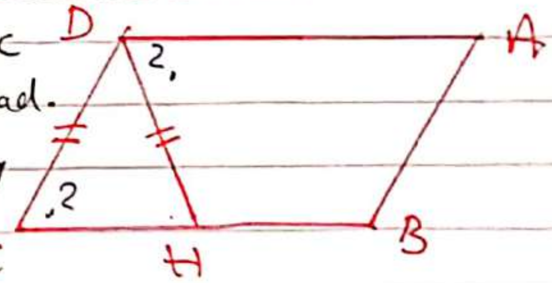
from ① and ②

$\therefore m(\angle ADE)$ exterior $= m(\angle EFD)$ interior

$\therefore$ BDEF is a cyclic quadrilateral.

ABCD is a parallelogram, $DH = DC$

prove that ① ABHD is cyclic quad.
② $\overline{AD}$ is a tangent of circle passing through triangle DHC.



Solution

$\because$ ABCD is a parallelogram

$\therefore m(\angle C) = m(\angle A) \rightarrow$ ①

In Δ DHC: $\because DH = DC$

$\therefore m(\angle C) = m(\angle DHC) \rightarrow$ ②

from ① and ②

$\therefore m(\angle DHC)$ exterior $= m(\angle A)$

$\therefore$ ABHD is a cyclic quadrilateral.

$\because \overline{AD} \parallel \overline{BC}$

$\therefore m(\angle ADH) = m(\angle DHC)$ alternate.

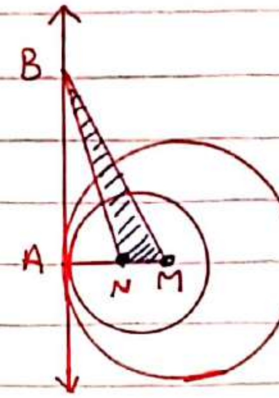
$\because m(\angle C) = m(\angle DHC)$

$\therefore m(\angle ADH) = m(\angle C)$

$\therefore \overline{AD}$ is a tangent.

P/ أحمد عمر

M and N are two circles whose radii lengths are 10 cm and 6 cm and touching internally at A, $\overleftrightarrow{AB}$ is a common tangent at A. If the area of $\triangle BMN = 24\ cm^2$ find length of $\overline{AB}$



Solution:

$\because$ the two circles touching internally

$\therefore MN = r_1 - r_2 = 10 - 6 = 4$ cm

$\because \overleftrightarrow{AB}$ is a common tangent

$\therefore \overleftrightarrow{MN} \perp \overleftrightarrow{AB}$

area of $\triangle BMN = \frac{1}{2} MN \times AB$

$\Rightarrow \frac{1}{2} \times 4 \times AB = 24 \Rightarrow 2AB = 24 \Rightarrow$ **AB = 12 cm**

م/ أحمد عمر

ABCD is a quadrilateral in which, AB = AD, $m(\angle ABD) = 30°$ and $m(\angle C) = 60°$

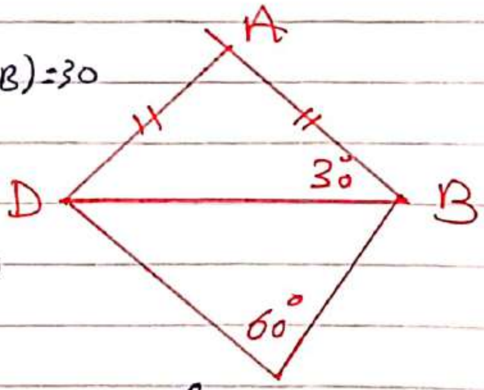
prove that: ABCD is a cyclic quadrilateral.

Solution:

In $\triangle ABD$

$\because AB = AD \quad \therefore m(\angle ABD) = m(\angle ADB) = 30$

$\therefore m(\angle A) = 180 - (30 + 30) = 120$



$\because m(\angle A) + m(\angle C) = 60 + 120 = 180$

and they are opposite

$\therefore$ ABCD is a cyclic quadrilateral.

مع أطيب الأمنيات بالنجاح والتفوق الباهر

٢٠٢١/٥/١٧ م/ أحمد عمر